

On the Axiomatization of Elgesem's Logic of Ability and Agency

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We investigate the semantics of the modal logic of agency and ability (*LAA*) proposed by Elgesem [1]. *LAA* is a classical bi-modal logic with the following axioms:

$$\text{T } EA \rightarrow A$$

$$\text{C } EA \wedge EB \rightarrow E(A \wedge B)$$

$$\text{EC } EA \rightarrow CA$$

$$\text{NoT } \neg C\top$$

where E and C are, respectively, the modal operators of agency and ability. For the semantics Elgesem adopts selection function models $\mathcal{S} = \langle W, f, v \rangle$, where W is a set of possible worlds, f is a function from $W \times 2^W$ to 2^W , and v is a valuation function. The modal operators are evaluated by the following clauses

$$w \models EA \text{ iff } w \in f(w, |A|) \quad w \models CA \text{ iff } f(w, |A|) \neq \emptyset$$

where $|A| = \{w : w \models A\}$. Moreover f satisfies the following conditions:

- $f(w, X) \subseteq X$;
- $f(w, X) \cap f(w, Y) \subseteq f(w, X \cap Y)$;
- $f(w, W) = \emptyset$.

It is immediate to see that $\neg C\perp$ is valid in the above class of models. We propose a class of neighbourhood models $\langle W, N^C, N^E, v \rangle$ where N^C and N^E are functions from $W \times 2^W$ to 2^{2^W} such that N^E is closed under intersection, $\forall w \forall X \in N_w^E (w \in X)$, $\forall w (W \notin N_w^C)$ and $N^E \subseteq N^C$. We prove that this class of models characterises *LAA*, but $\neg C\perp$ is not valid. Hence *LAA* is incomplete wrt the intended selection function semantics. We show how to modify the selection function semantics to regain completeness. We point out that the

resulting semantics relies on non-normal worlds. Accordingly we argue that an alternative semantics can be given in terms of multi-relation Kripke models with non-normal worlds. Finally we discuss some philosophical issues about the interpretation and appropriateness of the three types of semantics.

References

- [1] Dag Elgesem. The modal logic of agency. *Nordic Journal of Philosophical Logic*, 2(2):1–46, 1997.