On the Axiomatization of Elgesem's Logic of Ability and Agency

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We investigate the semantics of the modal logic of agency and ability (LAA) proposed by Elgesem [1]. LAA is a classical bi-modal logic with the following axioms:

T $EA \to A$

C $EA \wedge EB \to E(A \wedge B)$

EC $EA \to CA$

NoT $\neg C\top$

where E and C are, respectively, the modal operators of agency and ability. For the semantics Elgesem adopts selection function models $\mathcal{S} = \langle W, f, v\rangle$, where W is a set of possible worlds, f is a function from $W \times 2^W$ to 2^W, and v is a valuation function. The modal operators are evaluated by the following clauses

$$w \models EA \text{ iff } w \in f(w, |A|) \qquad w \models CA \text{ iff } f(w, |A| \neq \emptyset)$$

where $|A| = \{w : w \models A\}$. Moreover f satisfies the following conditions:

- $f(w, X) \subseteq X$;
- $f(w, X) \cap f(w, Y) \subseteq f(w, X \cap Y)$;
- $f(w, W) = \emptyset$.

It is immediate to see that $\neg C\bot$ is valid in the above class of models. We propose a class of neighbourhood models $\langle W, N^C, N^E, v\rangle$ where N^C and N^E are functions from $W \times 2^W$ to 2^{2^W} such that N^E is closed under intersection, $\forall w \forall X \in N^E_w(w \in X)$, $\forall w(W \notin N^C_w)$ and $N^E \subseteq N^C$. We prove that this class of models characterises LAA, but $\neg C\bot$ is not valid. Hence LAA is incomplete wrt the intended selection function semantics. We show how to modify the selection function semantics to regain completeness. We point out that the

resulting semantics relies on non-normal worlds. Accordingly we argue that an alternative semantics can be given in terms of multi-relation Kripke models with non-normal worlds. Finally we discuss some philosophical issues about the interpretation and appropriateness of the three types of semantics.

References

[1] Dag Elgesem. The modal logic of agency. *Nordic Journal of Philosphical Logic*, 2(2):1–46, 1997.