Semantics for Modelling Reason-based Preferences

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Abstract. In [13] the authors developed a logical system based on the definition of a new non-classical connective $\otimes$ originally capturing the notion of reparative obligation. The operator $\otimes$ and the system were proved to be appropriate for rather handling well-known contrary-to-duty paradoxes. Later on, a suitable model-theoretic possible-world semantics has been developed [4, 5]. In this paper we show how a version of this semantics can be used to develop a sound and complete logic of preference and offer a suitable possible-world semantics. The semantics is a sequence-based non-normal one extending and generalising semantics for classical modal logics.

1 Introduction

Theoretical and computational research in social choice theory is now recognised as relevant and is well-established in the MAS community. Indeed, it deals with the problem of how to aggregate in MAS individual preferences into a social or collective preference in order to achieve a rational collective decision [11].

Preliminarily to any useful contribution in this area we need to develop suitable formalisms and reasoning methods to represent and handle agents' preferences. In the current literature, we can find several approaches, among which the most remarkable in computational social choice theory are perhaps the following [2]:

- conditional preference networks, or CP-nets [1];
- prioritised goals [7, 17].

The second approach uses logical formalisms to describe the goals of the agents whose preferences are modelled as propositional formulae. This allows for a manageable and purely qualitative representation of preferences. Very recently, a new proposal in this perspective has been advanced [16], which presents a modal logic where a binary operator is meant to syntactically express preference orderings between formulae: each formula of this logic determines a preference ordering over alternatives based on the priorities over properties that the formula express. The authors recall that such types of formalisms are in fact capable of representing not just orderings over alternatives but the reasons that lead to the preferences [18]. The formalism is then interestingly used in [16] to originally treat the problem of collective choice in MAS as aggregation of logical formulae. The logic in [16] is clearly inspired by the work in [3], which in turn has a number of similarities with a system that was independently developed in [13] and where a Gentzen system was proposed in a different but related area—deontic logic—to reason about orderings on obligations. The idea that reasoning about preferences is crucial in deontic logic was introduced in semantic settings long time ago [15] (for recent discussions, [14, 21]). [13] is however based on the syntactic introduction of the new non-classical operator $\otimes$: the reading of an expression like $a \otimes b \otimes c$ is that a is primarily obligatory, but if this obligation is violated, the secondary obligation is b, and, if the secondary (CTD) obligation b is violated as well, then c is obligatory. These constructions can be used as well to reason about preferences. Thus, following the approach in [13], let $\Vdash$ be a non-classical consequence relation used to characterise conditional preferences. An expression like

$$Resident \Vdash \neg Pay\_Taxes \otimes \neg Pay\_Interest \otimes Pay\_Minimum$$

can be intuitively viewed as a conditional preference statement meaning the following:

1. if I'm resident in Italy, i.e. if *Resident* is the case, then not paying taxes is my actual preference, but,
2. if it happens that I pay taxes, then my actual preference is rather not to pay any interest, but
3. if I pay any interest, then my actual preference is pay a minimum.

Very recently, we have also devised a new semantics for $\otimes$ logics, which extends neighbourhood models with sequences of truth sets [4, 5]. In this paper we take advantage of our previous work and offer some technical results for a new preference logic. Our intent is to take the token from [16] and go deeper into semantically investigating such modal logics.

The layout the paper is as follows. Section 2 presents the basic logical system for $\otimes$ to represent and reason about preferences. The logic recalls some intuitions from [13, 4, 5]. Section 3 defines a sequence neighbourhood semantics suitable for the system which adjusts the one proposed in [4, 5]. Sections 5 and 6 provide, respectively, soundness and completeness results. Some conclusions end the paper.

2 A Logic for Reason-based Preferences

Let us present in this section a new variant of the logic presented in [13, 4, 5], a logic which was originally devised for modelling deontic reasoning and which is here revised to reason about preferences. The language consists of a countable set of atomic formulae. Well-formed-formulae are then defined using the usual Boolean connectives and the n-ary connective $\otimes$, which is intended to syntactically formalise preference ordering among reasons. The language also includes the modal operator Pr denoting the actual preferred reason or state of affairs: in other words, $\mathsf{Pr}p$ means that p is preferred. The interpretation of an expression $a \otimes b$ is that b is a the most preferred reason or state of affairs, but, if a is not the case then b is preferred.

Let $\mathscr{L}$ be a language consisting of a countable set of propositional letters $Prop = \{p_1, p_2, \dots\}$, the propositional constant $\bot$, round brackets, the boolean connective $\rightarrow$, the unary operator Pr, and a set of n-ary operators $\otimes^n$ for $n \in \mathbb{N}$, $n > 0$.

Definition 1 (Well Formed Formulae). *Well formed formulae (wffs) are defined as follows:*

– *Any propositional letter* $p \in Prop$ *and* $\bot$ *are wffs;*
– *If* a *and* b *are wffs, then* $a \to b$ *is a wff;*
– *If* a *is a wff and no operator* $\otimes^n$ *and* Pr *occurs in* a*, then* $\mathsf{Pr}a$ *is a wff;*
– *If* $a_1, \ldots, a_n$ *are wffs and no operator* $\otimes^n$ *and* Pr *occurs in any of them, then* $a_1 \otimes^n \cdots \otimes^n a_n$ *is a wff, where* $1 \leq n$*;*[3]
– *Nothing else is a wff.*

We use WFF to denote the set of well formed formulas.

Other Boolean operators are defined in the standard way, in particular $\neg a =_{def} a \to \bot$ and $\top =_{def} \bot \to \bot$.

We say that any formula $a_1 \otimes^n \cdots \otimes^n a_n$ is an $\otimes$-*chain*. The formation rules allow us to have $\otimes$-chain of any (finite) length, and the arity of the operator is equal to number of elements in the chain; accordingly we drop the index m from $\otimes^m$. Moreover, we will often use the prefix notation $\bigotimes_{i=j}^{n} a_i$ for $a_j \otimes \cdots \otimes a_n$. In addition we use the following notation: $\bigotimes_{i=j}^{n} a_i \otimes b \otimes \bigotimes_{k=l}^{m} c_k$, where $i, j \in \{0, 1\}$. The "a" part and "c" part are optional, i.e., they are empty when $i = 0$ or $j = 0$, respectively. Otherwise the expression stands for the following chain of $n+1+m$ elements: $a_1 \otimes \cdots \otimes a_n \otimes b \otimes c_1 \otimes \cdots \otimes c_m$

Let us define a Gentzen-style sequent calculus for $\otimes$.

Definition 2 (Sequents). *Let* $\vdash$ *and* $\Vdash$ *be two binary consequence relations defined over* $\mathscr{P}(WFF) \times WFF$*. Thus expressions* $\Gamma \vdash a$ *and* $\Gamma \Vdash a$ *are sequents where* Γ *is a finite (and possibly empty) set of wffs, and* a *is a wff.*

We use $\vdash$ for the consequence relation of classical propositional logic (see [10] for an appropriate set of rules), and $\Vdash$ for the consequence relation for the preference logic of $\otimes$. The following axiom and rules define the sequent calculus $\mathsf{E}^{\otimes}$ for $\Vdash$:

$$\Gamma, a \Vdash a \tag{ID}$$

This axiom allows us to use assumptions in $\Vdash$.

$$\frac{\vdash a}{\Gamma \Vdash a} \tag{PC}$$

The rule above allows us to import classical consequences in $\Vdash$.

$$\frac{\Gamma \Vdash a \qquad \Delta \Vdash a \to b}{\Gamma, \Delta \Vdash b} \tag{MP}$$

The combination of (ID), (PC) and (MP) enables us to use the full power of classical propositional logic in the right-hand side of the preference consequence relation.

$$\frac{\Gamma, a \Vdash b \qquad \Delta \Vdash a}{\Gamma, \Delta \Vdash b} \tag{Cut}$$

$$\frac{\Vdash a_j \equiv a_k}{\Gamma \Vdash (\bigotimes_{i=1}^{n} a_i) \equiv (\bigotimes_{i=1}^{k-1} a_i) \otimes (\bigotimes_{i=k+1}^{n} a_i)} \quad \text{(where } j < k\text{)} \tag{$\otimes$-shortening}$$

[3] We will use the prefix form $\otimes^1 a$ for the case of $n = 1$.

$$\frac{\Gamma \Vdash \bigotimes_{k=0}^{p} a_k \otimes (\bigotimes_{i=1}^{n} b) \otimes \bigotimes_{l=0}^{q} c_l \qquad \Delta, \{\neg b_1, \dots, \neg b_n\} \Vdash \bigotimes_{j=1}^{m} d_j}{\Gamma, \Delta, \Vdash \bigotimes_{k=0}^{p} a_k \otimes (\bigotimes_{i=1}^{n} b) \otimes \bigotimes_{j=1}^{m} d_j} \qquad (\otimes\text{-I})$$

$$\frac{\Gamma \Vdash (\bigotimes_{i=0}^{n} b_i) \otimes c \otimes \bigotimes_{j=0}^{m} d_j \qquad \Delta \Vdash \bigwedge_{i=0}^{n} \neg b_i}{\Gamma, \Delta \Vdash \mathsf{Pr} c} \qquad (\mathsf{Pr}\text{-detachment})$$

$$\frac{\Gamma \Vdash \mathsf{Pr} a}{\Gamma \Vdash \neg \mathsf{Pr} \neg a} \qquad (\otimes\text{-}\mathbf{D})$$

$$\frac{\Gamma \Vdash \bigotimes_{i=1}^{n} a_i}{\Gamma \Vdash \bigotimes_{i=1}^{n-1} a_i} \quad (\text{where } n > 1) \qquad (\otimes\text{-}\bot)$$

$$\frac{\Vdash b \equiv c}{\Gamma \Vdash (\bigotimes_{i=0}^{n} a_i \otimes b \otimes \bigotimes_{j=0}^{m} d_j) \equiv (\bigotimes_{i=0}^{n} a_i \otimes c \otimes \bigotimes_{j=0}^{m} d_j)} \qquad (\otimes\text{-}\mathbf{RE})$$

A few comments are in order.

The rule ($\otimes$-shortening) corresponds to duplication and contraction[4]: for example, $a \otimes b \otimes a$ is equivalent to $a \otimes b$. Intuitively, if I prefer not to get any damage, but if this happens I prefer to be compensated, and, if the damage is not compensated, then I prefer not to get any damage, this just means that my primary preference is not to get any damage and my secondary preference is to be compensated.

(Pr-detachment) is nothing but a rule allowing for detaching actual preferences from $\otimes$-chains, i.e, those preferences that hold in a given context. They reflect the intuitive reading of the $\otimes$ operator. Indeed, if $a \otimes b$, the primary preference should hold, and, if a is factually false ($\neg a$), then b must be preferred, i.e., $\mathsf{Pr} b$.

($\otimes$-I) is a peculiar introduction rule for $\otimes$. Let us illustrate ($\otimes$-I) by considering a simple instance of it as applied to a concrete example:

$$\frac{\Vdash \neg Pay\_Taxes \otimes \neg Pay\_Interest \qquad Pay\_Taxes \wedge Pay\_Interest \Vdash \otimes Pay\_Minimum}{\Vdash \neg Pay\_Taxes \otimes \neg Pay\_Interest \otimes Pay\_Minimum}$$

The sequent of the left-hand side states that my primary preference is not to pay taxes, but if this happens then my preference is not pay any interest (for example, by paying them in due time without delay). The sequent of the right-hand side rather states that, if I pay taxes and pay with interest (e.g., because I was late), then my preference is to pay the minimum amount. Hence, ($\otimes$-I) states that there is a chain of preferences dealing iteratively with the fact that my primary preference (not to pay any taxes) is not satisfied.

Schemata ($\otimes$-**D**) and ($\otimes$-$\bot$) ensure respectively internal and external consistency of preferences, similarly as in standard modal logic [6]. ($\otimes$-**D**) is nothing but a simple generalisation of modal **D** [6], stating that it is not possible to have both $\mathsf{Pr} a$ and $\mathsf{Pr} \neg a$, indeed we have the following derivation:

[4]Contraction in a logical sense, which is different from the one usually adopted in preference theory and which is captured by the subsequent derived rule ($\otimes$-contraction) (see Section 4). For this reason, we prefer to use in this first case the term "shortening".

$$\dfrac{\dfrac{\Gamma \Vdash \mathsf{Pr}a}{\Gamma \Vdash \neg\mathsf{Pr}\neg a} \qquad \Delta \Vdash \mathsf{Pr}\neg a}{\Gamma, \Delta \Vdash \bot}$$

The logic of $\otimes$ inherits the standard consistency rule (i.e., derive $\Gamma, \Delta \Vdash \bot$ from $\Gamma \Vdash a$ and $\Delta \Vdash \neg a$) from the underlying classical propositional consequence relation $\vdash$. This clearly holds for any $\otimes$-chain. Given the preference chain $n =\Vdash a \otimes b \otimes c$—meaning that a is preferred, and the second best preference is b, and the third best preferred one is c—asserting that n does not hold, i.e., $\Vdash \neg(a \otimes b \otimes c)$ amounts to a contradiction. But what about if we just assert that b is not the second best preference with respect to a, or that a is not actually preferred without having $\neg a$? These two cases are subsumed by n, thus they should result in a contradiction as well. ($\otimes$-$\bot$) ensure this effect by allowing us to derive all the initial (starting from the leftmost element) sub-chains of an existing $\otimes$-chain. In other words, if

$$\Vdash \neg \mathit{Pay\_Taxes} \otimes \neg \mathit{Pay\_Interest} \otimes \mathit{Pay\_Minimum}$$

then we can conclude that the following hold, too:

$$\Vdash \neg \mathit{Pay\_Taxes} \otimes \neg \mathit{Pay\_Interest}$$
$$\Vdash \neg \mathit{Pay\_Taxes}.$$

Finally, it should be intuitively clear that ($\otimes$-**RE**) generalises for $\otimes$-formulae the weakest inference rule for modal logics, i.e., the closure of $\Box$ (here Pr) under logical equivalence [6].

3 Sequence Semantics

Let us introduce the semantic structures that we use to interpret $\otimes$-formulas. In fact, they are just an extension of neighbourhood frames for classical modal logics.

Definition 3. *A* sequence frame *is a tuple $\mathscr{F} = \langle W, \mathscr{C} \rangle$ where:*

- *W is a non empty set of worlds;*
- *$\mathscr{C}$ is a neighbourhood function with the following signature*[5]

$$\mathscr{C} : W \mapsto 2^{((2^W)^n)} \qquad \textit{for } n \in \mathbb{N}.$$

In general, a sequence frame is nothing but a structure where the standard neighbourhood function is replaced by a function that establishes an order between elements (i.e., sets of worlds) of each neighbourhood associated to every world. Figure 1 offers a pictorial representation of the intuition.

The following definitions introduce the notion of redundancy and the operations of *zipping* and *s-zipping*, i.e., operations that, respectively, remove repetitions or redundancies occurring in $\otimes$-chains and in sequences of sets of worlds. Intuitively, these operations are necessary because, despite the fact the language allows for building expressions like $a \otimes b \otimes a$, these must be semantically evaluated using the sequences of sets of worlds $\langle \|a\|_V, \|b\|_V \rangle$ (see rule ($\otimes$-shortening)).

[5] As done sometimes with the standard neighbourhood function, we use the notation $\mathscr{C}_w$ to denote $\mathscr{C}(w)$.

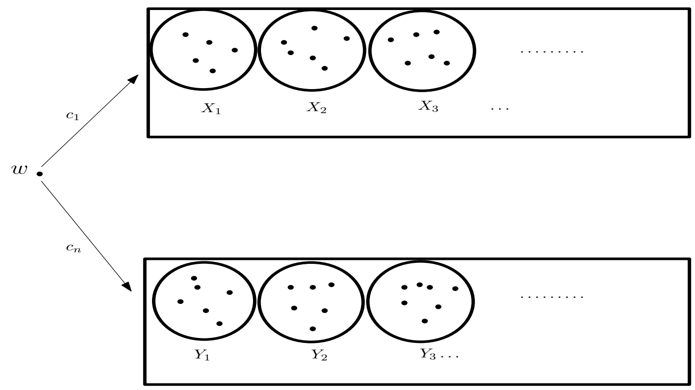

Fig. 1: Sequence basic structure: $X_1, X_2, X_3, \dots \subseteq W$ and $Y_1, Y_2, Y_3, \dots \subseteq W$

Definition 4. *A formula A is* redundant *iff $A = \bigotimes_{i=1}^{n} a_i$, $n > 1$ and $\exists a_j, a_k$, $1 \leq j,k \leq n$, $j \neq k$, such that $a_j \equiv a_k$.*

Definition 5. *Let $A = \bigotimes_{i=1}^{n} a_i$ be any redundant formula. We say that the non-redundant B is* zipped from *A iff B is obtained from A by applying recursively the operations below:*

1. *If $n = 2$, i.e., $A = a_1 \otimes a_2$, and $a_1 \equiv a_2$, then B, the* zipped from*, is $\mathsf{Pr} a_1$;*
2. *Otherwise, if $n > 2$, then for $1 \leq k \leq n$, if $a_j \equiv a_k$, for $j < k$, delete $\otimes a_k$ from the sequence.*

Let $X = \langle X_1, \dots, X_n \rangle$ be such that $X_i \in 2^W$ $(1 \leq i \leq n)$. We analogously say that Y is s-zipped from *X iff Y is obtained from X by applying the operations below:*

1. *If $n = 2$ and $X_1 = X_2$, then its* s-zipped from *Y is $\langle X_1 \rangle$;*
2. *Otherwise, if $n > 2$, then for $1 \leq k \leq n$, if $X_j = X_k$, for $j < k$, delete X_k from the sequence.*

Definition 6 (Models with sequences and truth of formulae). *A model $\mathscr{M}$ is a pair $\langle \mathscr{F}, V \rangle$ where $\mathscr{F}$ is a frame and V is a valuation such that:*

- *for any non-redundant $\bigotimes_{i=1}^{n} a_i$, $\models_w^V \bigotimes_{i=1}^{n} a_i$ iff there is a $c_j \in \mathscr{C}_w$ such that $c_j = \langle \|a_1\|_V, \dots, \|a_n\|_V \rangle$;*
- *for any redundant $\bigotimes_{i=1}^{n} a_i$, $\models_w^V \bigotimes_{i=1}^{n} a_i$ iff*
  - *$\bigotimes_{f=1}^{k} a_f$ is zipped from $\bigotimes_{i=1}^{n} a_i$, and*
  - *$\models_w^V \bigotimes_{f=1}^{k} a_f$.*
- *$\models_w^V \mathsf{Pr} a$ iff there is a $c_l \in \mathscr{C}_w$ such that:*
  - *$c_l = \langle \|a_1\|_V, \dots, \|a_n\|_V \rangle$;*
  - *for some $k \leq n$, $\|a_k\|_V = \|a\|_V$;*
  - *for $1 \leq j < k$, $w \notin \|a_j\|_V$.*

Figure 2 pictorially illustrates the types of models used for evaluating $\otimes$-formulae. In fact, we use only finite sequences, of sets of worlds, closed under s-zipping. A formula $\bigotimes_{i=1}^{n} a_i$ is true iff the corresponding appropriate finite sequence of sets of worlds (without redundancies) is in $\mathscr{C}_w$. Notice that the evaluation clause for $\mathsf{P}a$ works using sequences of length 1 or with longer sequences whenever a is the k's element of the $\otimes$-chain and the previous a_j are such that $w \notin \|a_j\|_V$, i.e., the previous preferences have not been satisfied in w.

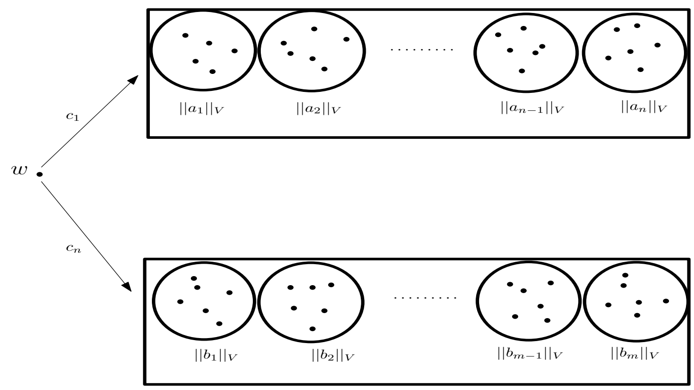

Fig. 2: Sequence models where finite sequences are used to evaluate the formulae $\bigotimes_{i=1}^{n} a_i$, $\dots$, $\bigotimes_{i=1}^{m} b_i$

Definition 7 (Truth of sequents). *Let $\Gamma \Vdash a$ be any sequent. Then,*

$$\models_w^V \Gamma \Vdash a \text{ iff, if } \forall w' \in W \text{ such that } \models_{w'}^V \Gamma, \text{ then } \models_{w'}^V a.$$

4 Choice Consistency: Contraction and Expansion

It is almost standard in social choice theory to assume two rationality conditions of choice (which are related with the fact that a choice function is rationalisable) [11]: *contraction consistency* and *expansion consistency*. The former one is "is concerned with keeping a chosen alternative choosable as the set is expanded by adding alternatives dominated [...] in other choices", while the latter one "is concerned with keeping a chosen alternative choosable as the set is contracted by dropping other alternatives" [20, page 65]. More precisely, contraction states that if an agent chooses some alternative from a set S of alternatives and this alternative remains available in a subset S' of S, then the agent chooses it from S'. Expansion somehow works in the opposite direction and requires that, given two sets S and S' of alternatives such that $S \subseteq S'$, for all pairs of alternatives in S, if one agent chooses them from S, then the agent still chooses from S' both of them or does not choose none of them. [11]. Although it has been argued that one possibility, among others, to avoid Arrow's impossibility result is precisely is to relax one of those principles [11, 2], these last are usually taken as basic standards of rationality in choice theory.

Notice that such conditions hold as well in the framework proposed in [16], where a simple semantic formulation is proposed, but no syntactic formalisation is given. Our logic, too, satisfies both conditions and a simple formalisation is possible.

Let us begin by considering contraction. Within our formalism, choices are ordered via the $\otimes$ operator, while a simple way to select arbitrary sets of alternatives is done by arbitrarily considering propositional formulae in the antecedents of $\Vdash$-sequents. Hence, contraction can be easily represented as follows:

$$\frac{a \Vdash \bigotimes_{i=1}^{n} b_i \qquad c \vdash a}{c \Vdash \bigotimes_{i=1}^{n} b_i} \quad (\otimes\text{-contraction})$$

($\otimes$-contraction) is clearly a derived rule, as it corresponds in our setting to logical monotonicity with respect to $\Vdash$. In fact, suppose that any state where c holds is also a state where a holds:

$$\frac{\dfrac{c \vdash a}{c \Vdash a} \qquad a \Vdash \bigotimes_{i=1}^{n} b_i}{c \Vdash \bigotimes_{i=1}^{n} b_i}$$

From the semantic point of view, it is plain to see that ($\otimes$-contraction) rule does not require any specific frame condition because, by construction, if, for all worlds $w \in \|a\|_V$ and $\models_w^V \bigotimes_{i=1}^{n} b_i$, since $\|c\|_V \subseteq \|a\|_V$ then for all $v \in \|c\|_V$ we trivially have that $\models_v^V \bigotimes_{i=1}^{n} b_i$.

The formulation of expansion is intuitive as well:

$$\frac{a \Vdash \bigotimes_{i=1}^{n} b_i \qquad c \Vdash \bigotimes_{i=1}^{n} b_i}{\Vdash ((a \vee d) \to \bigotimes_{i=1}^{n} b_i) \equiv ((c \vee d) \to \bigotimes_{i=1}^{n} b_i)} \quad (\otimes\text{-expansion})$$

Here, pairs of alternatives (more generally, pairs of sets of alternatives) are selected by assuming the truth of a and c and we state that a certain choice from $\bigotimes_{i=1}^{n} b_i$ is considered in both alternatives. Now, if pick up larger sets (which are determined by disjunctively adding any arbitrary propositional formula d), then either the same choice is preserved or it is abandoned in both alternatives.

Indeed, ($\otimes$-expansion) holds, since inference rule (PC) allows us to import in $\Vdash$-logic all consequences of classical logic[6]. The same idea can be easily checked in our sequence semantics. Indeed, it is plain to see that ($\otimes$-expansion) rule does not require, too, any specific frame condition: if, for any worlds $w \in \|a\|_V$ and $v \in \|c\|_V$ we have that $\models_w^V \bigotimes_{i=1}^{n} b_i$ and $\models_v^V \bigotimes_{i=1}^{n} b_i$, since $\|a\|_V \subseteq \|a \vee d\|_V$ and $\|c\|_V \subseteq \|c \vee d\|_V$, then, by simple set-theoretic considerations, for all worlds $w' \in \|a \vee d\|_V$ and $v' \in \|c \vee d\|_V$ we have either (i) $\models_{w'}^V \bigotimes_{i=1}^{n} b_i$ and $\models_{v'}^V \bigotimes_{i=1}^{n} b_i$, or (ii) $\not\models_{w'}^V \bigotimes_{i=1}^{n} b_i$ and $\not\models_{v'}^V \bigotimes_{i=1}^{n} b_i$.

[6]The proof is a rather long, cumbersome, but in fact a routine exercise in sequent calculi for classical propositional logic and is omitted. Just notice that $((a \to b) \wedge (c \to b)) \to (((a \vee d) \to b) \equiv ((c \vee d) \to b))$ is a tautology.

5 Soundness Results

5.1 System $\mathsf{E}^{\otimes}$

Let us prove in this section soundness results for the rules of system $\mathsf{E}^{\otimes}$, which consists of the following rules: ($\otimes$-shortening), (Pr-detachment), ($\otimes$-**RE**), ($\otimes$-I), ($\otimes$-**D**), ($\otimes$-$\perp$).

Lemma 1. ($\otimes$-**RE**) *is valid in the class of all sequence frames.*

Proof. The result for ($\otimes$-RE) trivially follows from the fact that the valuation clause for any $\otimes$-formula $\bigotimes_{i=1}^{n} a_i$, at any world w and with any valuation V, requires the existence of a sequence $c \in \mathscr{C}_w$ of truth sets $\langle \|a_1\|_V, \dots, \|a_n\|_V \rangle$. Then since for any i, $\|a_i\|_V = \|b_i\|_V$ ($a_i \equiv b_i$ for any frame and any valuation by assumption) there is also a sequence $\langle \|b_1\|_V, \dots, \|b_n\|_V \rangle \in \mathscr{C}_w$.

Also ($\otimes$-shortening) holds in general:

Lemma 2. ($\otimes$-shortening) *is valid in the class of all sequence frames.*

Proof. The proof follows directly from the valuation clause of redundant formulae, and from the definition of *redundancy*, *zipping*, and *s-zipping*.

Lemma 3. (Pr-detachment) *is valid in the class of all sequence frames.*

Proof. The proof trivially follows from the valuation clause for the operator Pr.

As we have shown, ($\otimes$-contraction rule) is a derived rule and does not need any specific investigation. In fact, it is plain to semantically see by construction that, if, for all $w \in Y$ and $\models_w^V \bigotimes_{i=1}^{n} a_i$ and $w \in Y$, if $X \subseteq Y$ then for all $v \in X$ we have $\models_v^V \bigotimes_{i=1}^{n} a_i$.

Let us now study additional rules that are not validated the class of all sequence frames. Let us first consider the introduction rule for $\otimes$, which requires extra semantic conditions.

Definition 8. *Let $\mathscr{F} = \langle W, \mathscr{C} \rangle$ be a frame. We say that $\mathscr{F}$ is* $\otimes$-extended *iff for any $w \in W$ and $c_i = \langle X_1, \dots, X_n \rangle \in \mathscr{C}_w$, if $\exists i, j$, such that $i \leq j \leq n$ and $\forall k$, $i \leq k \leq j$, $w \in W - X_k$, then $\langle Y_1, \dots, Y_m \rangle \in \mathscr{C}_w$, then there exists $c' \in \mathscr{C}_w$ such that c' is s-zipped from $\langle X_1, \dots, X_j, Y_1, \dots, Y_m \rangle$.*

Lemma 4. ($\otimes$-I) *is valid in the class of $\otimes$-extended sequence frames.*

Proof. Let us assume for simplicity that all the formulae are zipped. Suppose, for reductio, that $\otimes$-I does not hold in an $\otimes$-extended frame. Thus there is a world w such that

$$\models_w^V \bigwedge \Gamma \wedge \bigwedge \Delta \qquad \not\models_w^V \bigotimes_{k=0}^{p} a_k \otimes (\bigotimes_{i=1}^{n} b_i) \otimes \bigotimes_{j=1}^{m} d_j. \tag{1}$$

This means that

$$\langle \|a_0\|_V, \dots, \|a_p\|_V, \|b_1\|_V, \dots, \|b_n\|_V, \|d_1\|_V, \dots, \|d_m\|_V \rangle \notin \mathscr{C}_w. \tag{2}$$

From the assumption the premises of $\otimes$-I are true in w, and $\models_w^V \bigwedge \Gamma$, thus $\models_w^V \bigotimes_{k=0}^{p} a_k \otimes (\bigotimes_{i=1}^{n} b_i) \otimes c$. Hence

$$\langle \|a_0\|_V, \ldots, \|a_p\|_V, \|b_1\|_V, \ldots, \|b_n\|_V, \|c\|_V \rangle \in \mathscr{C}_w. \tag{3}$$

Suppose that $w \in \|\neg b_i\|$ (for $1 \leq i \leq n$); therefore $\models_w^V \bigwedge_{i=1}^{n} \neg b_i$. In addition, again from the assumption, $\models_w^V \bigwedge \Delta \wedge \bigwedge_{i=1}^{n} \neg b_i$. From this and the truth of the second premise of $\otimes$-I in w we obtain that $\models_w^V \bigotimes_{j=1}^{m} d_j$, which means that

$$\langle \|d_1\|_V, \ldots, \|d_m\|_V \rangle \in \mathscr{C}_w \tag{4}$$

on the hypothesis that $w \in \|\neg b_i\|$ for $1 \leq i \leq n$. Since the frame is $\otimes$-extended

$$\langle \|a_0\|_V, \ldots, \|a_p\|_V, \|b_1\|_V, \ldots, \|b_n\|_V, \|d_1\|_V, \ldots, \|d_m\|_V \rangle \in \mathscr{C}_w \tag{5}$$

which contradicts (2).

Let us consider rules for ensuring consistency, i.e., ($\otimes$-**D**) and ($\otimes$-$\perp$).

Definition 9 (Seriality). *Let $\mathscr{F} = \langle W, \mathscr{C} \rangle$ be a frame. We say that $\mathscr{F}$ is* serial *iff $\forall w \in W$ and $\forall c_i = \langle X_1, \ldots, X_n \rangle \in \mathscr{C}_w$, there is no $c_j = \langle Y_1, \ldots, Y_m \rangle \in \mathscr{C}_w$ such that $Y_j = W - X_k$ if for all X_g, Y_h, $g < k \leq n$, $h < j \leq m$, $w \notin X_g, w \notin Y_h$.*

Lemma 5. ($\otimes$-**D**) *is valid in the class of serial sequence frames.*

Proof. The proof is straightforward. Consider any arbitrary sequent $\Gamma \Vdash \mathsf{Pr}a$ and suppose there is a serial frame $\mathscr{F}$, a valuation V, and a world w such that

$$\models_w^V \Gamma \Vdash \mathsf{Pr}a \tag{6}$$

and

$$\not\models_w^V \Gamma \Vdash \neg\mathsf{Pr}\neg a. \tag{7}$$

Hence, there is a world v such that $\models_v^V \mathsf{Pr}a$ and $\not\models_v^V \neg\mathsf{Pr}\neg a$. By the valuation clause for Pr, this implies that

1. there exists an $\otimes$-chain $\bigotimes_{j=1}^{m} d_j$ true at v where (a) $d_1 = a$, or (b) $d_h = a$, $1 < h \leq m$, and $\models_v^V \Vdash \bigwedge_{l=1}^{h-1} \neg d_l$,
2. there exists an $\otimes$-chain $\bigotimes_{i=1}^{n} b_i$ true at v where (i) $b_1 = \neg a$, or (ii) $b_k = \neg a$, $1 < k \leq n$, and $\models_v^V \Vdash \bigwedge_{j=1}^{h-1} \neg b_j$.

From 1 we have that $\langle \|a\|_V, \|d_2\|_V, \ldots, \|d_m\|_V \rangle \in \mathscr{C}_v$ or $\langle \|d_1\|_V, \ldots, \|d_{h-1}\|_V, \|a\|_V, \|d_{h+1}\|_V, \ldots, \|d_m\|_V \rangle \in \mathscr{C}_v$ and $v \notin \|d_1\|_V, \ldots, \|d_{k-1}\|_V$.

From 2 we have that $\langle W - \|a\|_V, \|b_2\|_V, \ldots, \|b_n\|_V \rangle \in \mathscr{C}_v$ or $\langle \|b_1\|_V, \ldots, \|b_{k-1}\|_V, W - \|a\|_V, \|b_{k+1}\|_V, \ldots, \|b_n\|_V \rangle \in \mathscr{C}_v$ and $v \notin \|b_1\|_V, \ldots, \|b_{k-1}\|_V$.

Hence, $\mathscr{F}$ is not serial, thus leading to a contradiction.

Definition 10 ($\otimes$-seriality). *Let $\mathscr{F} = \langle W, \mathscr{C} \rangle$ be a frame. We say that $\mathscr{F}$ is* $\otimes$-serial *iff for any $w \in W$ and for any finite sequence $c_i = \langle X_1, \ldots, X_n \rangle \in \mathscr{C}_w$, $n > 1$, the sequence $\langle X_1, \ldots, X_{n-1} \rangle \in \mathscr{C}_w$.*

Lemma 6. ($\otimes$-$\bot$) *is valid in the class of* $\otimes$*-serial sequence frames.*

Proof. The proof is straightforward. Consider any arbitrary sequent $\Gamma \Vdash \bigotimes_{i=1}^{n} a_i$ and suppose there is an $\otimes$-serial frame $\mathscr{F}$, a valuation V, and a world w such that

$$\models_w^V \Gamma \Vdash \bigotimes_{i=1}^{n} a_i \tag{8}$$

and

$$\not\models_w^V \Gamma \Vdash \bigotimes_{i=1}^{n-1} a_i. \tag{9}$$

Hence, there is a world v such that $\models_v^V \bigotimes_{i=1}^{n} a_i$ and $\not\models_v^V \bigotimes_{i=1}^{n-1} a_i$. By the valuation clause for $\otimes$-chains, this implies that

1. there exists a sequence $\langle \|a_1\|_V, \ldots, \|a_n\|_V \rangle \in \mathscr{C}_v$, and
2. there is no sequence $\langle \|a_1\|_V, \ldots, \|a_{n-1}\|_V \rangle \in \mathscr{C}_v$.

Thus, $\mathscr{F}$ is not $\otimes$-serial, contrary to the assumption.

6 Semantic Completeness

6.1 Completeness of $\mathsf{E}^{\otimes}$

In this section we shall provide a semantic completeness theorem via canonical models for $\mathsf{E}^{\otimes}$, as defined in Section 2.

Definition 11. *Two sequents* $\Gamma \Vdash a$ *and* $\Delta \Vdash b$ *are* inconsistent *if and only if* $\Gamma \cup \Delta$ *is a consistent set (i.e.,* $\Gamma, \Delta \not\Vdash \bot$*) and* $\Vdash a \wedge b \rightarrow \bot$.

Let us start by constructing the worlds of a canonical model:

1. Let W^C be the set of all possible maximal consistent sets of formulae in the language of $\mathsf{E}^{\otimes}$, constructed with a standard Lindenbaum procedure.
2. Take any $w \in W^C$. Let:
   (a) $w_0^+ := w$;
   (b) Let $\Gamma_1, \Gamma_2, \ldots$ be an enumeration of all the possible sequents in the language (where $Cl(v)$ is the closure of v under all the rules of the logic). Set $w_{n+1}^+ := Cl(w_n^+ \cup \{\Gamma_n\})$ if $\bigcap W^C \cup \{\Gamma_n\}$ is consistent; $w_{n+1}^+ := w_n^+$ otherwise.
   (c) $w^+ := \bigcup_{n \in \mathbb{N}} w_n^+$.
3. Set $W^+ := \{w^+ \mid w \in W^C\}$

Notice that clause (2b) of this construction guarantees that the set of sequents is the same for any v^+.

Definition 12 **($\mathsf{E}^{\otimes}$-Canonical Models).** *A sequence model with sequences* $\mathscr{M} := \langle W^+, \mathscr{C}, V \rangle$ *is a canonical model for* $\mathsf{E}^{\otimes}$ *if and only if:*

1. *For any propositional letter $p \in Prop$, $\|p\|_V := |p|_{\mathsf{E}^\otimes}$, where $|p|_{\mathsf{E}^\otimes} := \{w \in W^+ \mid p \in w\}$*
2. *Let $\mathscr{C} := \bigcup_{w \in W} \mathscr{C}_w$, where for each $w \in W$, $\mathscr{C}_w := \{\langle \|a_1\|_V, \ldots, \|a_n\|_V \rangle \mid a_1 \otimes \cdots \otimes a_n \in w\} \cup \{\langle \|a\| \rangle_V \mid \mathsf{Pr}a \in w\}$, where each a_i is a meta-variable for a Boolean formula and $a_1 \otimes \cdots \otimes a_n$ is* zipped.

Lemma 7 (Truth Lemma). *For any $w \in W^+$ and for any formula or sequent A, $A \in w$ if and only if $\models^V_w A$.*

Proof. Given the construction of the canonical model, this proof is easy and can be given by induction on the length of an expression A. We consider only a few relevant cases.

Assume A has the form $a_1 \otimes \cdots \otimes a_n$ and is redundant (clearly the case for non redundant formulae is easier and does not need to be considered here). Suppose $a_i \otimes \cdots \otimes a_n \in w$. Then, by $\otimes$-shortening, we have that the formula $b_1 \otimes \cdots \otimes b_j$, the *zipped* form of A, is also in w. By definition of canonical model we have that there is a sequence $\langle \|b_1\|_V, \ldots, \|b_j\|_V \rangle \in \mathscr{C}_w$. Following from the semantic clauses given to evaluate $\otimes$-formulae, it holds that $\models^V_w a_1 \otimes \cdots \otimes a_n$.

Now suppose that $\models^V_w a_1 \otimes \cdots \otimes a_n$. By definition, there is a zipped formula $b_1 \otimes \cdots \otimes b_j$ such that $\models^V_w b_1 \otimes \cdots \otimes b_j$. Thus, C_w contains an ordered j-tuple $\langle \|b_1\|_V, \ldots, \|b_j\|_V \rangle$. By definition of $\mathscr{C}_w$ it follows that $b_1 \otimes \cdots \otimes b_j \in w$ and by $\otimes$-shortening, all the *unzipped* forms of $b_1 \otimes \cdots \otimes b_j$ are also in w, including $a_1 \otimes \cdots \otimes a_n$.

If, on the other hand, A has the form $\mathsf{Pr}b$ and $\mathsf{Pr}b \in w$, then $\langle \|b\|_V \rangle \in \mathscr{C}_w$ and, by definition $\models^V_w \mathsf{Pr}b$. Conversely, if $\models^V_w \mathsf{Pr}b$, then there is an s-zipped sequence $\langle \|c_0\|_V, \ldots, \|c_n\|_V, \|b\|_V, \|d_1\|_V, \ldots, \|d_m\|_V \rangle \in \mathscr{C}_w$ and for $0 \leq i \leq n$, $w \notin \|c\|_i$. Thus, since any c_i is Boolean and w is maximal, $\neg c_0, \ldots, \neg c_n \in w$. Moreover $\bigotimes_{i=0}^n c_i \otimes b \otimes \bigotimes_{j=1}^m d_j \in w$. Hence by the Pr-detachment rule, $\mathsf{Pr}b \in w$.

If A is a sequent $\Gamma \Vdash a$ belonging to w, take any $v^+ \in W^+$ s.t. $\models^V_{v^+} \bigwedge \Gamma$. By induction hypothesis, $\Gamma \subseteq v^+$. By construction of v^+, $\Gamma \Vdash a \in v^+$, hence $a \in v^+$ and by induction hypothesis $\models^V_{v^+} a$. Thus, $\models^V_{w^+} \Gamma \Vdash a$. Conversely, assume $\Gamma \Vdash a \notin w^+$. By construction of w^+ it means that Γ is consistent with $\bigcap W^C$, i.e., $\Gamma \subseteq \bigcap W^C$ and $\Gamma \subseteq w^+$. By induction hypothesis, $\models^V_{w^+} \bigwedge \Gamma$. Also, $a \wedge b \to \bot$ for some $b \in \bigcap W^C$, hence $a \notin w^+$ and by induction hypothesis $\not\models^V_{w^+} a$.

For any sequent or formula A that is not derivable in $\mathsf{E}^\otimes$ it holds that $A \notin \bigcap W^+$ and hence for any $w^+ \in W^+$, $\not\models^V_{w^+} A$ by Lemma 7.

Lemma 8. *The canonical frame for $\mathsf{E}^\otimes$ is:*

1. *$\otimes$-Extended (as in Definition 8);*
2. *Serial (as in Definition 9);*
3. *$\otimes$-Serial (as in Definition 10).*

Proof. The proof is straightforward.

1. Consider a world w^+ such that (i) $\bigotimes_{i=0}^p a_i \otimes \bigotimes_{j=1}^n b_j \otimes \bigotimes_{l=0}^q c_l \in w^+$ and (ii) $\neg b_1, \ldots, \neg b_n \Vdash \bigotimes_{k=1}^m d_k \in w^+$. From (i) by Lemma 7

$$\models^V_{w^+} \bigotimes_{i=0}^{p} a_i \otimes \bigotimes_{j=1}^{n} b_j \otimes \bigotimes_{l=0}^{q} c_l \tag{10}$$

and thus

$$\langle \|a_0\|_V, \ldots, \|a_p\|_V, \|b_1\|_V, \ldots, \|b_n\|_V, \|c_0\|_V, \ldots, \|c_q\|_V \rangle \in \mathscr{C}_{w^+}. \tag{11}$$

Suppose $w^+ \in W - \|b_i\|$ for $1 \leq i \leq n$. Again by Lemma 7, $\neg b_i \in w^+$ (for $1 \leq i \leq n$), then from (ii) and the construction of w^+, $\bigotimes_{k=1}^{m} d_k \in w^+$, and $\bigotimes_{i=0}^{p} a_i \otimes \bigotimes_{j=1}^{n} b_j \otimes \bigotimes_{k=1}^{m} d_k \in w^+$. Thus, by Lemma 7,

$$\models_{w^+}^{V} \bigotimes_{k=1}^{m} d_k \tag{12}$$

$$\models_{w^+}^{V} \bigotimes_{i=0}^{p} a_i \otimes \bigotimes_{j=1}^{n} b_j \otimes \bigotimes_{k=1}^{m} d_k \tag{13}$$

which means

$$\langle \|d_1\|_V, \ldots, \|d_m\|_V \rangle \in \mathscr{C}_{w^+} \tag{14}$$

$$\langle \|a_0\|_V, \ldots, \|a_p\|_V, \|b_1\|_V, \ldots, \|b_n\|_V, \|d_1\|_V, \ldots, \|d_m\|_V \rangle \in \mathscr{C}_{w^+}. \tag{15}$$

which show that the canonical model is $\otimes$-extended.

2. If, for reductio, there are a world w^+ and two sequences belonging to $\mathscr{C}_{w^+}$

$$\langle \|a_1\|_V, \ldots, \|a_m\|_V, \|b\|_V, \|c_1\|_V, \ldots, \|c_m\|_V \rangle$$
$$\langle \|d_1\|_V, \ldots, \|d_j\|_V, -\|b\|_V, \|e_1\|_V, \ldots, \|e_k\|_V \rangle,$$

$w^+ \notin \|a_i\|_V$ for each i and $w^+ \notin \|d_i\|_V$ for each i, it would follow that both $\mathsf{Pr}a$, and $\mathsf{Pr}\neg a$ belong to w^+, although by $\otimes$-D $\neg\mathsf{Pr}\neg a \in w^+$.

3. The proof for $\otimes$-seriality is trivial and follows directly from the construction of the canonical model and the presence of $\otimes$-$\bot$.

Corollary 1. *The logic $\mathsf{E}^{\otimes}$ is sound and complete with respect to the class of sequence frames that are extended, serial, and $\otimes$-serial.*

7 Conclusion and Related Work

This paper offered a semantic study of the $\otimes$ operator originally introduced in [13] to model deontic reasoning and contrary-to-duty obligations. We showed that a suitable Gentzen-style sequent calculus incorporating $\otimes$-expressions can be characterised in a class of structures extending neighbourhood frames with sequences of sets of worlds. We argued that the formalism and the semantics can be employed, with some adjustments, to grasp various forms of reasoning about reason-based preferences. In this perspective, our contribution may offer useful insights for establishing connections between the proof-theoretic and model theoretic approaches to preference reasoning. Also, we showed that the logic validates both Contraction and Expansion Consistency [11, 16], thus satisfying two basic rationality conditions in social choice theory.

The current logic falls within the research on prioritised goals [7, 17], i.e., on formalisms for describing the goals of the agents whose preferences are modelled as propositional formulae. This allows for a purely qualitative representation of preferences. Before the recent developments in MAS [2], the most extensive (and, still the most advanced) work on preferences was done in the context of deontic logic. A first line of inquiry was mainly semantic-based: deontic sentences are interpreted in settings with ideality orderings on possible worlds or states [15]. This approach is quite flexible: depending on the properties of the preference or ideality relation, different deontic logics can be obtained. This semantic approach has been fruitfully renewed in the '90 for example by [19, 22], and most recently by works such as [14, 21], which have confirmed the vitality of this line of inquiry. The second line was proof-theoretic: in this second area, the Gentzen system proposed in [13] was definitely seminal for us in developing the current proposal. [13] is based on the introduction of the non-classical binary operator $\otimes$: the reading of an expression like $a \otimes b$ is that a is primarily obligatory, but is this obligation is violated, the secondary obligation is b. Inference rules introduced by [13]—in particular, ($\otimes$-shortening) and ($\otimes$-I)—are proposed here, too.

In the context of preference logics several proposals can be mentioned [7, 17, 18]. However, two works have specifically inspired our effort: [16] and [3]. [16] is very recent and presents a modal logic where a binary operator is meant to syntactically express preference orderings between formulae: each formula of this logic determines a preference ordering over alternatives based on the priorities over properties that the formula express. While the formalism is interesting in that it can represent not just orderings over alternatives but the reasons that lead to the preferences [18], the modal logic for expressing individual preferences is in fact equivalent to **S5**, which amounts to being a very strong and simple option (indeed, the main concern in this work is preference aggregation): as we argued, weaker but very expressive logics can be adopted. The qualitative choice logic (QCL) of [3] is a propositional logic for representing alternative, ranked options for problem solutions, using a substructural ordered disjunction. It offers a much richer alternative with respect to [16], showing a number of similarities with [13] (the two formalisms have been developed independently) and the one discussed here. A major differences is, for instance, that the $\otimes$-detachment produces conclusions that are modalised and not just factual. The semantics and proof theory of [3], though based on similar intuitions, are however technically different from ours: semantics is based on the degree of satisfaction of a formula in a particular (classical) model. Consequences of QCL theories can be computed through a compilation to stratified knowledge bases which in turn can be compiled to classical propositional theories. The consequence relation of [3] satisfies properties usually considered intended in nonmonotonic reasoning, such as cautious monotonicity and cumulative transitivity.

The preference operator $\otimes$ has been combined with Defeasible Logic to provide a computationally oriented approach to modelling alternative goals of rational agents to then select plans [8, 9]. More recently [12] investigates different forms of $\otimes$ detachment to identify different types of goal like mental attitudes for agents.

A number of open research issues are left for future work. Among others, we plan to explore decidability questions using, for example, the filtration methods. The fact that neighbourhoods contain sequences of sets of worlds instead of sets is not expected to make the task significantly harder than the one in standard neighbourhood semantics for modal logics.

Second, we expect to enrich the language and allow for nesting of $\otimes$-expressions, thus having formulae like $a \otimes \neg(b \otimes c) \otimes d$. We argued in [13] that the meaning of those formulae is not clear in deontic reasoning. However, a semantic analysis of them in the sequence semantics can clarify the issue. Indeed, in the current language we can evaluate in any world w formulae like $\neg(a \otimes b)$, which semantically means that there is no sequence $\langle \|a\|_V, \|b\|_V \rangle \in \mathscr{C}_w$. Conceptually, expressions like that may express meta-preferences, i.e., preferences about preference orderings. However, this reading poses interesting conceptual and technical problems.

Finally, we plan to apply the our framework to social choice theory by checking how our analysis impacts on the collective choice rules proposed in [16].

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