Justice Delayed Is Justice Denied: Logics for a Temporal Account of Reparations and Legal Compliance

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Abstract. In this paper we extend the logic of violation proposed by [14] with time, more precisely, we temporalise that logic. The resulting system allows us to capture many subtleties of the concept of legal compliance. In particular, the formal characterisation of compliance can handle different types of legal obligation and different temporal constraints over them. The logic is also able to represent, and reason about, chains of reparative obligations, since in many cases the fulfillment of these types of obligation still amount to legally acceptable situations.

1 Introduction

Developments in open MAS have pointed out that normative concepts can play a crucial role in modeling agents' interaction [24,8]. Like in human societies, desirable properties of MASs can be ensured if the interaction of artificial agents adopts institutional models whose goal is to regiment agents' behaviour through normative systems in supporting coordination, cooperation and decision-making. However, to keep agents autonomous it is often suggested that norms should not simply work as hard constraints, but rather as soft constraints [4]. In this sense, norms should not limit in advance agents' behaviour, but would instead provide standards which can be violated, even though any violations should result in sanctions or other normative effects applying to non-compliant agents. The detection of violations and the design of agents' compliance can amount to a relatively affordable operation when we have to check whether agents are compliant with respect to simple normative systems. But things are tremendously harder when we deal with realistic, large and articulated systems of norms such as the law. To the best of our knowledge, no systematic investigation has been so far proposed in this regard in the MAS field.

Among other things, the complexities behind the concept of legal compliance are due to the following reasons:

Reparative Obligations Legal norms often specify obligatory actions to be taken in case of their violation. Obligations in force after some other obligations have been violated correspond in general to contrary-to-duty obligations (CTDs) (see [7] for an overview).

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A peculiar subclass of CTDs is particularly relevant for the law: the so-called reparative obligations. For instance, in contract and in tort law reparative obligations protect individual legitimate interests by imposing actions that compensate any damages following from non-compliance [12]. These constructions affect the formal characterisation of legal compliance since they identify situations that are not ideal, but still legally acceptable. Consider the following example (where norms have as usual a conditional structure: if the antecedents are jointly the case, then the consequent is obligatory):

$$\begin{aligned}&\mathit{Invoice} \Rightarrow \mathrm{OBL}\mathit{PayBy7days}\\&\quad \mathrm{OBL}\mathit{PayBy7days}, \neg \mathit{PayBy7days} \Rightarrow \mathrm{OBL}\mathit{Pay5\%Interest}\\&\qquad \mathrm{OBL}\mathit{Pay5\%Interest}, \neg \mathit{Pay5\%Interest} \Rightarrow \mathrm{OBL}\mathit{Pay10\%Interest}\end{aligned}$$

What about if a customer violates both the obligation to pay by 7 days after having received the invoice for her purchase, and the obligation to pay the 5% of interest of the due amount, but she pays the total amount plus the 10% of interest? In the legal perspective (which aims at protecting the rights of the vendor), the customer is compliant.

If so, these constructions can give rise to very complex rule dependencies, because we can have that the violation of a single rule can activate other (reparative) rules, which, in case of their violation, refer to other rules, and so forth [15]. Clearly, if we take the above legal norms in isolation, the depicted situation is non-compliant, since two applicable legal norms are violated. However, if we compensate the violations, then we are still in a "legal" situation.

Obligation and Time The law makes use of different types of obligations (see Section 2) also depending on how legal effects are temporally qualified. A first basic distinction is between those legal obligations which persist over time unless some other and subsequent events terminate them (e.g., "If one causes damage, one has to provide compensation"), and those that hold at a specific time on the condition that the norm preconditions hold and with a specific temporal relationship between such preconditions and the obligation (e.g., "If one is in a public building, one is forbidden to smoke").

In regard to the concept of compliance, it is worth noting that we may have obligations requiring (1) to be always fulfilled during a certain time interval, (2) that a certain condition must occur at least once before a certain deadline and such that the obligations may, or may not, persist after this deadline if they are not complied with, (3) that something is done immediately [13].

Things are definitely harder when these types of obligations occur in chains of reparative obligations. For example, if the primary obligation is persistent and states to pay before tomorrow, and the secondary (reparative) obligation is to pay a fine in three days after the violation of the primary obligation, we are compliant not only when we pay by tomorrow, but also when we do not meet this deadline and pay both the due amount and the fine on the day after tomorrow.

Formal Requirements for Legal Compliance From a logical point of view, a formal characterisation of the concept of legal compliance requires to address the following related research tasks: (a) We need a logic able to handle different types of legal obligation and different temporal constraints over them; (b) This logic should be able to represent, and reason about, chains of reparative obligations. In particular, we need a procedure for making hidden conditions and reparative chains explicit; without this, we do not know whether a certain situation is legally acceptable; (c) We have to embed into the logic aspects of time, such as persistence and deadlines.

In the following section we informally discuss the types of obligation we will handle in the proposed framework.

2 The Many Faces of Obligations

We can distinguish *achievement* from *maintenance obligations* [13]. For an *achievement obligation*, a certain condition must occur at least once before a deadline:

Example 1. Customers must pay within 7 days, after receiving the invoice.

The deadline refers to an obligation triggered by receipt of the invoice. After that the customer is obliged to pay. The fulfilment of the obligation by its deadline terminates the persistence of the obligation.

For *maintenance obligations*, a certain condition must obtain during all instants before the deadline:

Example 2. After opening a bank account, customers must keep a positive balance for 30 days.

In Example 2 the deadline only signals that the obligation is terminated: a violation occurs when the obliged state does not obtain at some time before the deadline.

Finally, *punctual obligations* only apply to single instants:

Example 3. When banks proceed with any wire transfer, they must transmit a message, via SWIFT, to the receiving bank requesting that the payment is made according to the instructions given.

Punctual obligations apply to single instants; they can be thought as maintenance obligations in force in time intervals where the endpoints are equal. Typically punctual obligations must occur at the same time of their triggering conditions.

Norms can be associated with an explicit sanction. For example,

Example 4. Customers must pay within 7 days, after receiving the invoice. Otherwise, 10% of interest must be paid within 10 days.

Example 5. After opening a bank account, customers must keep a positive balance for 30 days. Otherwise, their account must be immediately blocked.

A sanction is often implemented through a separate obligation, which is triggered by a detected violation. Thus, different types of obligations can be combined in chains of reparative obligations: in Example 4, the violation of the primary achievement obligation is supposed to be repaired by another achievement obligation; in Example 5, the violation of a primary maintenance obligation is compensated by a punctual obligation.

We introduced in [15,14] the non-boolean connective $\otimes$: a formula like $a \otimes b$ means that a is obligatory, but if the obligation a is not fulfilled, then the obligation b is activated and becomes in force until it is satisfied or violated. However, the violation condition of an obligation varies depending on the types of obligations used. In the remainder, we will extend the approach of [15,14] by adding temporal qualifications to cover these cases.

3 Temporalised Violation Logic

To start with, we consider a logic whose language is defined as follows:

Definition 1 (Language). *Let $\mathscr{T} = (t_1, t_2, \dots)$ be a discrete linear order of instants of time, $Atm = \{a, b, \dots\}$ be a set of atomic propositions, and O be a deontic operator.*

- *A literal is either an atomic proposition or the negation of an atomic proposition, that is: $Lit = Atm \cup \{\neg l : l \in Atm\}$.*
- *If $l \in Lit$ and $t \in \mathscr{T}$, then l^t is a* temporal literal*; $\top$ and $\bot$ are* temporal literals. *$TLit$ denotes the set of temporal literals.*
- *If l^t is a temporal literal, then Ol^t and $\neg Ol^t$ are* deontic literals. *The set of deontic literals is denoted by $DLit$.*
- *If a^{t_a} and b^{t_b} are temporal literals, $t \in \mathscr{T}$, and $t_a \leq t$, then $a^{t_a} \otimes_t^x b^{t_b}$ (for $x \in \{p, m, a\}$) is an $\otimes$-chain.*
- *If α is an $\otimes$-chain, a^{t_a} is a temporal literal and $t \in \mathscr{T}$, then $\alpha \otimes_t^x a^{t_a}$ (for $x \in \{p, m, a\}$) is an $\otimes$-chain.*
- *Let α be either a temporal literal, or an $\otimes$-chain, $t \in \mathscr{T}$, then $\bot$, $\alpha \otimes \bot$ and $\alpha \otimes_t \bot$ are deontic expressions. Nothing else is a deontic expression. The set of deontic expressions is denoted by $DExp$.*

Let us explain the intuitive meaning of the various elements of the language. The meaning of a temporal literal a^t is that proposition a holds at time t. The deontic literal Oa^t means that we have the obligation that a holds at time t. The meaning of $\top$ and $\bot$ is that $\top$ is a proposition that is always complied with (or in other terms it is impossible to violate) and $\bot$, on the other hand, is a proposition that is always violated (or it is impossible to comply with). According to the intended meaning it is useless in the present context to temporalise them. $\otimes$ is a binary operator to express complex normative positions. More specifically, the meaning of a deontic expression like $\alpha \otimes_{t_\alpha}^x a^{t_a} \otimes_{t'_a}^y b^{t_b}$ is that the violation of a triggers a normative position whose content is b^{t_b}. What counts as a violation of a^{t_a} depends on the parameter x, encoding the type of obligation whose content is a, and the two temporal parameters t_a and t'_a. The nature of the normative position whose content is b^{t_b} depends on $\otimes^y$. The type of obligation whose content is a^{t_a} is determined by x. If $x = p$, then we have a punctual obligation (in this case we require that $t_a = t'_a$) and this means that to comply with this prescription have must hold at time t_a. If $x = a$, then we have an achievement obligation; in this case a is obligatory from t_a to t'_a, and the obligation is fulfilled if a holds for at least one instant of time in the interval $[t_a, t'_a]$. Finally, if $x = m$, similarly to the previous case, a is obligatory in the interval $[t_a, t'_a]$, but in this case, to comply with the prescription, a must hold for all the instants in the interval. As we have said, the $\otimes$ operator introduces normative positions in response to a violation of the formula on the left of the operator, thus this is a contrary-to-duty operator. An important application of contrary-to-duties is that a contrary-to-duty can be used to encode a sanction or compensation or reparation for a violation. The focus of this paper is mostly on this type of contrary-to-duties. What about $DExp$? The meaning of a $DExp$, in particular of $\perp$ at the end of them, is that we have reached a situation that cannot be compensated for, This means that the penultimate element of a deontic expression identifies the 'last chance' to be compliant. After that the deontic expression results in a situation that cannot be complied with anymore.

Definition 2 (Rules/norms[3]). *A* rule $r : \Gamma \hookrightarrow \alpha$ *is an expression where* r *is a unique rule label,* $\Gamma \subseteq TLit \cup DLit$*,* $\hookrightarrow \in \{\Rightarrow^x, \leadsto\}$*,* $\alpha \in DExp$*. If* $\hookrightarrow$ *is* $\Rightarrow^x$*, the rule is a* defeasible rule*; If* $\hookrightarrow$ *is* $\leadsto$*, the rule is a* defeater*. For defeasible rules* $x \in \{a, m, p\}$*, and: If* $x = a$ *the rule is an* achievement rule*; If* $x = m$ *the rule is a* maintenance rule*; If* $x = p$ *the rule is a* punctual rule*. For defeaters* $\alpha \in TLit$*.*

A rule is a relationships between a set of premises and a conclusion, thus we use several types of rules to describe different types of relationships. We use the distinction of the types of the rules (defeasible and defeater) for the strength of the relationship between the premises and the conclusion. The superscript x indicates the mode of a rule. The mode of a rule tells us what kind of conclusion we can obtain from the rule. In the context the mode identifies the type of obligation we can derive. The idea is that from a rule of mode a, an achievement rule, we derive an achievement obligation.

A defeasible rule is a rule where when the body holds then typically the conclusion holds too unless there are other rules/norms overriding it. For example, when you receive an invoice, you have the obligation to pay for it:

$$r_1 : invoice^t \Rightarrow^a pay^t \tag{1}$$

The meaning of the above rule is that if you received an invoice at time t, then you have the obligation to pay for it, starting from time t.[4]

Defeaters are the weakest rules. They cannot be used to derive obligations, but they can be used to prevent the derivation of an obligation. Hence, they can be used to describe exceptions to obligations, and in this perspective they can be used to terminate existing obligations. For this reason, the arrow $\leadsto$ is not labeled by either a, m, nor p. Continuing the previous example, paying for the invoice terminates the obligation to pay for it:

$$r_2 : paid^t \leadsto pay^t \tag{2}$$

Rule r_2 says that if you pay at time t then, from time t on, there is no longer the obligation to pay. Notice that the defeater does not introduce the prohibition to pay again.

Definition 3 (Defeasible Theory). *A* Defeasible Theory *is a structure* $(F, R, \succ)$*, where* F*, the set of facts, is a set of temporal literals;* R *is a set of rules; and* $\succ$*, the superiority relation, is a binary relation over* R*.*

[3] In the reminder, we will interchangeably use both the terms 'norm' and 'rule', but we will prefer 'norm' whenever the usage of the term 'rule' may be confused with 'inference rule'.

[4] We assume the usual inter-definability between obligations and prohibition, thus $O\neg \equiv F$, and $F\neg \equiv O$.

A theory corresponds to a normative system, i.e., a set of norms, where every norm is modelled by rules. The superiority relation is used for conflicting rules, i.e., rules whose conclusions are complementary literals, in case both rules fire. Notice that we do not impose any restriction on the superiority relation, which is a binary relation that just determines the relative strength of two rules. For example, if we consider the two rules in (1) and (2), given an invoice, and that the invoice has been paid the two rules alone cannot allow us to conclude anything due to the sceptical nature of Defeasible Logic. But if we further establish that $r_2 \succ r_1$, then the second rule prevails, and we will conclude that we are permitted not to pay.

Definition 4. *Given an $\otimes$-chain α, the* length *of α is the number of elements in it. Given an $\otimes$-chain $\alpha \otimes^x_t b^{t_b}$, the* index *of b^{t_b} is n iff the length of $\alpha \otimes^x_t b^{t_b}$ is n. We also say that b appears at index n in $\alpha \otimes^x_t b^{t_b}$.*

Definition 5 (Notation). *Given a rule $r : \Gamma \hookrightarrow \alpha$, we use $A(r) = \Gamma$ to indicate the antecedent or body of the rule, and $C(r) = \alpha$ for the consequent or conclusion or head of r. Given a set or rules R: $R_\Rightarrow$ is the set of defeasible rules in R; $R_\leadsto$ is the set of defeaters in R; R^a is the set of achievement rules in R; R^m is the set of maintenance rules in R; R^p is the set of punctual rules in R; $R[a^t]$ is the set of rules whose head contains a^t. $R[a^t,k]$ is the set of rules where a^t is at index k in the head of the rules.*

To simplify and uniform the notation we can combine the above notations, and we use subscripts and superscripts before the indication relative of the head. Thus, for example, $R_\leadsto[p^{10}]$ is the set of defeaters whose head is the temporal literal p^{10}, and the rule

$$r : a_1^{t_1} \dots, a_n^{t_n} \Rightarrow^p a^{10} \otimes^m_{10} b^{20}$$

is in $R^m_\Rightarrow[b^{20}]$, as well as in $R^p[a^{10}]$ and $R_\Rightarrow[b^{20},2]$.

Finally, notice that we will sometimes abuse the notation and omit (a) the timestamp t_l in the temporal literal l^{t_l} whenever it is irrelevant to refer to it in the specific context, (b) the mode x in the rule arrow $\Rightarrow^x$ when x can be instantiated with any of a, m or p, (c) x and y in $\otimes^x_y$ when x and y can be instantiated, respectively, with any of a, m, p and with any time instants.

Properties of the $\otimes$ operator When we have a deontic expression $\alpha = a_1 \otimes \cdots \otimes a_n$ we do not have information about the type of obligation for the first element. This information is provided when we use the expression in a rule. In this section we are going to investigate properties of $\otimes$, in particular when two (sub-)sequences of deontic expression are equivalent and thus we can replace them preserving the meaning of the whole expression (or rule). To simplify the notation, we introduce the following conventions.

Definition 6. *Let $r : \Gamma \Rightarrow^x \alpha$ be a rule, then ${}^x\alpha$ is an $\otimes$-sequence. The empty sequence is an $\otimes$-sequence. If $\alpha \otimes^x_{t_\alpha} a^{t_a} \otimes^y_{t'_a} \beta \otimes^z_{t_\beta} \gamma$ is an $\otimes$-sequence, where α, β, γ are $\otimes$-sequences, then ${}^x a^{t_a} \otimes^y_{t'_a} \beta$ is an $\otimes$-sequence.*

Given a rule $r : \Gamma \hookrightarrow^x \alpha \otimes^y_t \beta$, α can be the empty $\otimes$-sequence, and if so, then the rule reduces to $r : \Gamma \Rightarrow^y \beta$.

From now on, we will refer to $\otimes$-sequences simply as sequences and we will provide properties for sequences to be used in rules.

The first property we want to list is the commutativity of the $\otimes$ operator.

$$\alpha \otimes_t^x (\beta \otimes_{t'}^y \gamma) \equiv (\alpha \otimes_t^x \beta) \otimes_{t'}^y \gamma \tag{3}$$

We extend the language with $\top$ and $\bot$. Given their meaning, those two propositions can be defined in terms of the following sequence and equivalence[5]

$$^p a^0 \otimes_0^p \neg a^0 \equiv \top \qquad \bot \equiv \neg\top. \tag{4}$$

The two new propositions are useful to define reduction rules for deontic expressions. Let us start with equivalences for $\top$.

$$\top \otimes \alpha \equiv \top. \tag{5}$$

This equivalence says that a violation of $\top$ can be compensated by α; however, $\top$ is a proposition that cannot be violated. Thus, the whole expression cannot be violated. What about when $\top$ appears as the last element of $\otimes$?

$$\alpha \otimes \top \equiv \top. \tag{6}$$

The meaning of $\alpha \otimes \top$ is that $\top$ is the compensation of α, thus the violation of α is sanctioned by $\top$. This means that the violation of α is always compensated for, thus we have a norm whose violation does not result in any effective sanction, thus violating α does not produce any effect. Hence, we have two possibilities: to reject (6) if we are interested to keep trace of violations, or to accept it if we want to investigate the effects of violations. In this paper we take the first option and we reject the equivalence of $\alpha \otimes \top$ and $\top$. Notice that reducing $\alpha \otimes \top$ to α would change the meaning, since this would mean that the violation of α cannot be repaired. To see this we move to the properties involving $\bot$.

$$^p a^{t_a} \otimes_{t_a}^x \bot \equiv a^{t_a} \tag{7}$$

The above equivalence specifies that if $\bot$ is the compensation of a punctual obligation a at time t, then there is no compensation, since the compensation cannot be complied with. The effect of the rules is that we can eliminate $\bot$ from the deontic expression and we maintain the same meaning. Notice, however, that the same is not true for other types of obligations. For example, for $x \in \{a, m\}$, we cannot eliminate $\bot$ from rules like

$$\Gamma \Rightarrow^x a^t \otimes_{t'}^m \bot$$

since the resulting expression would be $\Gamma \Rightarrow^x a^t$ and we would miss the information about the deadline to comply with a. Nevertheless, the following equivalence states that $\bot$ can be safely eliminated if it is not the last element of a deontic expression, or when it is the 'compensation' of a maintenance obligation without deadline.

$$\alpha \otimes_{t_\alpha}^x \bot \otimes_t^y \beta \equiv \alpha \otimes_{t_\alpha}^y \beta \qquad ^m a^{t_a} \otimes \bot \equiv {}^m a^{t_a} \tag{8}$$

[5] In case one wants the temporalised version, $\top^t \equiv {}^p a^t \otimes_t^p \neg a^t$, and $\bot^t \equiv \neg\top^t$.

To complete the description for the properties for $\perp$, we need to specify when we can generate a new rule introducing $\perp$ from two other rules.

$$\frac{\Gamma \Rightarrow^x \alpha \otimes^y_{t_\alpha} a^t \otimes_{t_a} \perp \quad \Delta \hookrightarrow \neg a^{t'} \otimes_{t''} \perp}{\Gamma, \Delta \Rightarrow^x \alpha \otimes^y_{t_\alpha} a^t \otimes_{t'-1} \perp} t < t' \text{ and } y \in \{a, m\} \tag{9}$$

The meaning of the above inference rule is that if we have a norm determining the termination of an obligation, then we can encode the obligation, the time when the obligation comes to force and the time when the norm terminates its normative effect. The idea behind a norm like $a^t \Rightarrow^x b^{t'}$ is the obligation b enters into force from time t'. Here we assume the intuition developed in [16] that a 'new' rule takes precedence over a conclusion obtained in the past and carrying over to the current moment by persistence. Thus if we have a rule $c^{t_c} \Rightarrow \neg b^{t''}$ with $t'' > t'$ the rule for $\neg c^{t''}$ effectively terminates the force of the obligation b. Consider the following instance of the rule

$$\frac{r_1 : a^5 \Rightarrow^m b^{10} \otimes_{15} \perp \quad c^{12} \Rightarrow^a \neg b^{12} \otimes_{20} \perp}{a^5, c^{12} \Rightarrow^m b^{10} \otimes_{11} \perp}$$

In this case r_1 puts the obligation of b in force in the interval from 10 to 15, and r_2 enforces $\neg b$ from 12 to 20, thus when both conditions to apply, the effective time when the obligation of b is in force is from 10 to 11 (after that the obligation $\neg b$ enters into force).

The $\otimes$ operator, introduced in [14], is a substructural operator corresponding to the comma on the right hand side of a sequent in sequent system. In a classical sequent system both the left hand side and right hand side of a sequent are set of formulas, thus the order of the formulas does not matter, and properties like contraction and duplication hold. In [14] we established the equivalence between $\alpha \otimes a \otimes \beta \otimes a \otimes \gamma$ and $\alpha \otimes a \otimes \beta \otimes \gamma$. This states that if a literal occurs multiple times, we can remove all but the first occurrence. We turn our attention to study conditions under which we have contraction for the various (combination of) $\otimes$ operators we have.

Punctual-Punctual	${}^p a^t \otimes^x_t \beta \otimes^p_{t_\beta} a^{t'} \otimes^y_{t'} \gamma \equiv^p a^t \otimes^x_t \beta \otimes^p_{t_\beta} \perp \otimes^y \gamma$	$t = t'$
Punctual-Achievement	${}^p a^t \otimes^x_t \beta \otimes^a_{t_\beta} a^{t_s} \otimes^y_{t_e} \gamma \equiv^p a^t \otimes^x_t \beta \otimes^a_{t_\beta} \perp \otimes^y \gamma$	$t = t_s = t_e$
Punctual-Maintenance	${}^p a^t \otimes^x_t \beta \otimes^m_{t_\beta} a^{t_s} \otimes^y_{t_e} \gamma \equiv^p a^t \otimes^x_t \beta \otimes^m_{t_\beta} \perp \otimes^y \gamma$	$t \in [t_s, t_e]$
Achievement-Punctual	${}^a a^{t_s} \otimes^x_{t_e} \beta \otimes^p_{t_\beta} a^{t'} \otimes^y_{t'} \gamma \equiv^a a^{t_s} \otimes^x_{t_e} \beta \otimes^p_{t_\beta} \perp \otimes^y \gamma$	$t' = t_s = t_e$
Achievement-Achievement	${}^a a^{t_s} \otimes^x_{t_e} \beta \otimes^a_{t_\beta} a^{t'_s} \otimes^y_{t'_e} \gamma \equiv^a a^{t_s} \otimes^x_{t_e} \beta \otimes^a_{t_\beta} \perp \otimes^y \gamma$	$[t'_s, t'_e] \subseteq [t_s, t_e]$
Achievement-Maintenance	${}^a a^{t_s} \otimes^x_{t_e} \beta \otimes^m_{t_\beta} a^{t'_s} \otimes^y_{t'_e} \gamma \equiv^a a^{t_s} \otimes^x_{t_e} \beta \otimes^m_{t_\beta} \perp \otimes^y \gamma$	$[t_s, t_e] \cap [t'_s, t'_e] \neq \emptyset$
Maintenance-Punctual	${}^m a^{t_s} \otimes^x_{t_e} \beta \otimes^p_{t_\beta} a^{t'} \otimes^y_{t'} \gamma \equiv^m a^{t_s} \otimes^x_{t_e} \beta \otimes^p_{t_\beta} \perp \otimes^y \gamma$	$t' = t_s = t_e$
Maintenance-Achievement	${}^m a^{t_s} \otimes^x_{t_e} \beta \otimes^a_{t_\beta} a^{t'_s} \otimes^y_{t'_e} \gamma \equiv^m a^{t_s} \otimes^x_{t_e} \beta \otimes^a_{t_\beta} \perp \otimes^y \gamma$	$t_s = t_e = t'_s = t'_e$
Maintenance-Maintenance	${}^m a^{t_s} \otimes^x_{t_e} \beta \otimes^m_{t_\beta} a^{t'_s} \otimes^y_{t'_e} \gamma \equiv^m a^{t_s} \otimes^x_{t_e} \beta \otimes^m_{t_\beta} \perp \otimes^y \gamma$	$[t_s, t_e] \subseteq [t'_s, t_e]$

Table 1. Reductions to $\perp$

Punctual-Punctual	${}^{p}a^{t} \otimes_{t}^{x} \beta \otimes_{t_\beta}^{p} {\sim} a^{t'} \otimes_{t'}^{y} \gamma \equiv^{p} a^{t} \otimes_{t}^{x} \beta \otimes_{t_\beta}^{p} \top \otimes^{y} \gamma$	$t = t'$
Punctual-Achievement	${}^{p}a^{t} \otimes_{t}^{x} \beta \otimes_{t_\beta}^{a} {\sim} a^{t_s} \otimes_{t_e}^{y} \gamma \equiv^{p} a^{t} \otimes_{t}^{x} \beta \otimes_{t_\beta}^{a} \top \otimes^{y} \gamma$	$t \in [t_s, t_e]$
Punctual-Maintenance	${}^{p}a^{t} \otimes_{t}^{x} \beta \otimes_{t_\beta}^{m} {\sim} a^{t_s} \otimes_{t_e}^{y} \gamma \equiv^{p} a^{t} \otimes_{t}^{x} \beta \otimes_{t_\beta}^{m} \top \otimes^{y} \gamma$	$t = t_s = t_e$
Achievement-Punctual	${}^{a}a^{t_s} \otimes_{t_e}^{x} \beta \otimes_{t_\beta}^{p} {\sim} a^{t'} \otimes_{t'}^{y} \gamma \equiv^{a} a^{t_s} \otimes_{t_e}^{x} \beta \otimes_{t_\beta}^{p} \top \otimes^{y} \gamma$	$t' = t_s = t_e$
Achievement-Achievement	${}^{a}a^{t_s} \otimes_{t_e}^{x} \beta \otimes_{t_\beta}^{a} {\sim} a^{t'_s} \otimes_{t'_e}^{y} \gamma \equiv^{a} a^{t_s} \otimes_{t_e}^{x} \beta \otimes_{t_\beta}^{a} \top \otimes^{y} \gamma$	$[t_s, t_e] \subseteq [t'_s, t'_e]$
Achievement-Maintenance	${}^{a}a^{t_s} \otimes_{t_e}^{x} \beta \otimes_{t_\beta}^{m} {\sim} a^{t'_s} \otimes_{t'_e}^{y} \gamma \equiv^{a} a^{t_s} \otimes_{t_e}^{x} \beta \otimes_{t_\beta}^{m} \top \otimes^{y} \gamma$	$[t'_s, t'_e] \subseteq [t_s, t_e]$
Maintenance-Punctual	${}^{m}a^{t_s} \otimes_{t_e}^{x} \beta \otimes_{t_\beta}^{p} {\sim} a^{t'} \otimes_{t'}^{y} \gamma \equiv^{m} a^{t_s} \otimes_{t_e}^{x} \beta \otimes_{t_\beta}^{p} \top \otimes^{y} \gamma$	$t' = t_s = t_e$
Maintenance-Achievement	${}^{m}a^{t_s} \otimes_{t_e}^{x} \beta \otimes_{t_\beta}^{a} {\sim} a^{t'_s} \otimes_{t'_e}^{y} \gamma \equiv^{m} a^{t_s} \otimes_{t_e}^{x} \beta \otimes_{t_\beta}^{a} \top \otimes^{y} \gamma$	$[t_s, t_e] \subseteq [t'_s, t'_e]$
Maintenance-Maintenance	${}^{m}a^{t_s} \otimes_{t_e}^{x} \beta \otimes_{t_\beta}^{m} {\sim} a^{t'_s} \otimes_{t'_e}^{y} \gamma \equiv^{m} a^{t_s} \otimes_{t_e}^{x} \beta \otimes_{t_\beta}^{m} \top \otimes^{y} \gamma$	$t_s = t_e = t'_s = t_e$

Table 2. Reductions to $\top$

Tables 1 and 2 give the conditions to remove duplicates of the same atom. Consider for example, the instance ${}^{p}a^{10} \otimes^{m} a^{0} \otimes_{20} \bot$ of the reduction Punctual-Maintenance in Table 1, where the primary obligation is to have a at time 10, and whose compensation is to maintain a from 0 to 20. To trigger the secondary obligation we should have the violation of the primary obligation. This means that ${\sim}a$ holds at 10, but this implies that it is not possible to maintain a from 0 to 20, thus it is not possible to compensate the violation of the primary obligation. Notice that in several cases the reductions are possible only when the intervals are just single instants.

Introduction Rules Besides the properties given so far the full meaning of the $\otimes$ operator is given by the rules to introduce (and modify) the operator. The general idea of the introduction rules is to determine the conditions under which a norms is violated. If these conditions imply a particular obligation then, then this obligation can be seen as a compensation of the norm the conditions violate.

$$\frac{\Gamma \Rightarrow^{x} \alpha \otimes_{t_\alpha}^{p} b^{t_b} \otimes_{t_b}^{Y} \gamma \qquad \Delta, \neg b^{t_b} \hookrightarrow^{z} \delta}{\Gamma, \Delta \Rightarrow^{x} \alpha \otimes_{t_\alpha}^{p} b^{t_b} \otimes_{t_b}^{z} \delta} \otimes I_p$$

The punctual obligation $O^{p} b^{t_b}$ (implied by the first sequent) holds only at time t_b thus the only instant when the obligation can be violated is exactly t_b.

Rule $\otimes I_p$ is the standard rule to introduce a (novel) compensation or CTD (see [14] for further discussion about it).

$$\frac{\Gamma \Rightarrow^{x} \alpha \otimes_{t_\alpha}^{m} b^{t_s} \otimes_{t_e}^{y} \beta \qquad \Delta, \Theta \Rightarrow^{z} \delta}{\Gamma, \Delta \Rightarrow^{x} \alpha \otimes_{t_\alpha}^{m} b^{t'_s} \otimes_{t'_e}^{z} \delta} \otimes I_m \text{ where } \Theta = \{{\sim} b^{t'} : t_s < t'_s \leq t' \leq t'_e \leq t_e\}.$$

The introduction rule for $\otimes^{m}$ defines a slice of the interval where a specific compensation of the violation holds. This conditions requires a rule whose antecedent contains the complement of a maintenance obligation in the head of the other rule, such that the literal is temporalised with the last n consecutive instants. For example given the rules

$$a^{10} \Rightarrow^{m} b^{10} \otimes_{20} \bot \qquad c^{15}, \neg b^{17}, \neg b^{18}, \neg b^{19} \Rightarrow^{p} d^{20} \otimes_{20} \bot$$

we can derive the new rule

$$a^{10}, c^{15} \Rightarrow^{m} b^{17} \otimes^{p}_{19} d^{20} \otimes_{20} \bot$$

The conditions to derive a new compensation rule for an achievement obligation are more complicated. As we have seen from the previous two cases, the structure of the introduction rules is that the negation of a consequent of a norm is a member of the antecedent of another norm (with the appropriate time). This ensures that the antecedent of the norm is a breach of the other one. The idea is the same for achievement obligations, but now detecting a violation is more complex.

$$\frac{\Gamma \Rightarrow^{x} \alpha \otimes^{a}_{t_\alpha} a^{t^s_a} \otimes^{x}_{t^e_a} \beta \quad \Delta, Oa^{t'_a}, \sim a^{t'_a} \Rightarrow^{z} \delta \quad \{\Delta, \sim a^{t''_a} \Rightarrow^{z} \delta\}_{\forall t''_a : t^s_a < t'_a \leq t''_a \leq t^e_a}}{\Gamma, \Delta \Rightarrow^{x} \alpha \otimes^{a}_{t_\alpha} a^{t^s_a} \otimes^{z}_{t'_a} \delta} \otimes I_a$$

The idea behind the introduction of a compensation for achievement obligation is that we have to determine that the obligation has not been fulfilled at a time before the deadline and for all instant greater or equal to it the complement is required. Essentially, the $\otimes I_a$ amounts to shortening the deadline for an achievement obligation.

$$\frac{a^1 \Rightarrow^{a} b^5 \otimes_{10} \bot \quad Ob^8, \neg b^8 \Rightarrow^{p} c^{15} \otimes_{15} \bot \quad \neg b^9 \Rightarrow^{p} c^{15} \otimes_{15} \bot \quad \neg b^{10} \Rightarrow^{p} c^{15} \otimes_{15} \bot}{a^1 \Rightarrow^{a} b^5 \otimes^{p}_{8} c^{15} \otimes_{15} \bot}$$

The first norm initially sets the deadline by when b at to be achieved to 10. The last n norms, in this case $n = 2$, have as premises the opposite of an obligation of the first norm covering the last n instant of the force period of the obligation and the same conclusion. This means that refraining to fulfill the obligation in the last n instants results in the same consequence. The last part is to assess that we have a violation. This is achieved by the second norm; here, we have the obligation in the antecedent (an achievement obligation is no longer in force in two cases: we are after the deadline or the content of the obligation has been achieved), thus the condition Ob^8 and $\neg b^8$ is to ensure that the obligation is still in force at the time, and the combination of the norms ensures that from now on not fulfilling the obligation results in the same compensation.

Subsumption The inference rules combine premises in such a way as the deontic content of at least one of them is included by the conclusion. Consequently, some original rules are no longer needed. To deal with this issue we introduce the notion of subsumption. A norm subsumes a second when the behaviour of the second norm (its compliance condition) is implied by the first one. Here below is an example illustrating this idea.

Example 6. Consider the following norms:

$$r: \ Invoice^{t} \Rightarrow^{a} Pay^{t} \otimes^{p}_{t+6} PayInterest^{t+7} \otimes_{t+7} \bot$$
$$r': Invoice^{t}, OPay^{t+6}, \neg Pay^{t+6} \Rightarrow^{a} PayInterest^{t+7} \otimes_{t+8} \bot$$

The first norm says that after the seller sends the invoice, the buyer has the achievement obligation to pay within 7 days, otherwise immediately after the violation the buyer has to pay the principal plus the interest (punctual obligation to pay at $t+7$). According to the second norm, given the same set of circumstances *Invoice* at time t, if we have still the obligation on the seventh day after the invoice receipt date and the payment is not made yet, we have the achievement obligation to pay the interest by the eighth day. However, (a) the primary obligation of r' obtains when we have a violation of the primary obligation of r; (b) after the primary obligation of r is violated, complying with its secondary obligation entails complying with the primary obligation of r' (but not vice versa); (c) hence, r is more general than r', and so the latter can be discarded.

In what follows, Definition 10 characterizes the concept of subsumption that we have informally illustrated in Example 6. Since we need to check whether the compliance of a norm guarantees the compliance of another norm (the subsumed one), we provide below the following auxiliary definitions to establish (a) Definition 7: the modes with which the compliance conditions for one obligation covers the compliance conditions of another one; (b) Definition 8: when the compliance conditions of an $\otimes$-chain cover the compliance conditions of another $\otimes$-chain; (c) Definition 9: the conditions under which a literal belonging to an $\otimes$-chains is violated (indeed, subsumption allows to remove the norms whose applicability conditions require to violate another norm, while these conditions are encoded in the $\otimes$-chain of the subsuming norm).

Definition 7. *Let $X, Y \in \{a, m, p\}$. Then, $Y \sqsubseteq X$ iff 1) if $Y = a$, then $X \in \{a, m, p\}$; 2) if $Y = m$, then $X = m$; 3) if $Y = p$, then $X \in \{p, m\}$.*

Definition 8. *Let*

$$\gamma = {}^{x_1}c_1^{t_{c_1}} \otimes^{x_2}_{t'_{c_1}} c_2^{t_{c_2}} \otimes^{x_3}_{t'_{c_2}} \cdots \otimes^{x_j}_{t'_{c_{j-1}}} c_j^{t_{c_j}} \qquad \beta = {}^{y_1}b_1^{t_{b_1}} \otimes^{y_2}_{t'_{b_1}} b_2^{t_{b_2}} \otimes^{y_3}_{t'_{b_2}} \cdots \otimes^{y_k}_{t'_{b_{k-1}}} b_k^{t_{b_k}}$$

be $\otimes$-chains. The $\otimes$-chain γ d-includes *the $\otimes$-chain β iff*

1. $j = k$,
2. $c_i = b_i$,
3. $y_i \sqsubseteq x_i$;
4. (a) *if $y_i = a$, then $t'_{c_i} \geq t_{b_i}$ when $x_i = m$, otherwise $t_{c_i} = t_{b_i}$ and $t'_{c_i} \leq t'_{b_i}$;*
   (b) *if either $y_i = m$ or $y_i = p$, then $t_{c_i} \leq t_{b_i}$ and $t'_{c_i} \geq t'_{b_i}$*

where $1 \leq i \leq j, k$.

Definition 9. *Let ${}^{x_1}c_1^{t_{c_1}} \otimes^{x_2}_{t'_{c_1}} c_2^{t_{c_2}} \otimes^{x_3}_{t'_{c_2}} \cdots \otimes^{x_j}_{t'_{c_{j-1}}} c_j^{t_{c_j}}$ be any $\otimes$-chain. For any c_i, where $1 \leq i \leq j$, a set X* violates *c_i iff*

1. *if $x_i = a$, then $X = \{Oc_i^{t'_{c_i}}, \sim c_i^{t'_{c_i}}\}$;*
2. *if $x_i = m$ or $x_i = p$, then $X \subseteq \{\sim c_i^t | t_{c_i} \leq t \leq t'_{c_i}\}$.*

Definition 10. *Let $r_1 : \Gamma \Rightarrow \alpha \otimes \beta \otimes \gamma$ and $r_2 : \Delta \Rightarrow \delta$ be two rules, where α, β, γ, and δ are $\otimes$-chains such that $\gamma = {}^{z_1}c_1^{t_{c_1}} \otimes^{z_2}_{t'_{c_1}} c_2^{t_{c_2}} \otimes^{z_3}_{t'_{c_2}} \cdots \otimes^{z_l}_{t'_{c_{l-1}}} c_l^{t_{c_l}}$.*

Then r_1 subsumes *r_2 iff*

1. *$\Gamma = \Delta$ and α d-includes δ; or*
2. *$\Gamma \cup X = \Delta$, where X violates all elements in α, and β d-includes δ; or*
3. *$\Gamma \cup Y = \Delta$, where Y violates all elements in β, and $\alpha \otimes {}^{z_1}c_1^{t_{c_1}} \otimes^{z_2}_{t'_{c_1}} c_2^{t_{c_2}} \otimes^{z_3}_{t'_{c_2}} \cdots \otimes^{z_n}_{t'_{c_{n-1}}} c_n^{t_{c_n}}$ d-includes δ, where $n \leq l$.*

4 Proof Conditions

We introduce the conditions that allow us to determine whether an obligation is in force at time t (and the type of obligation as well). The problem reduces to determine whether a (temporalised) literal follows from a theory, in other terms whether we can derive the (temporalised) literal. In addition the conditions allow us to establish whether a theory has been complied with. In Definition 1 we stated that a deontic expression extends an $\otimes$-chain with $\bot$ at the end. Thus effectively the penultimate element of a deontic expression identifies the 'last chance' to be compliant. After that the deontic expression results in a situation that cannot be complied with anymore. Hence, checking whether a theory is not compliant amounts to deriving $\bot$.

Definition 11. *A tagged literal is an expression #l, where* $\# \in \{+\partial, -\partial, +\partial^p, -\partial^p, +\partial^a, -\partial^a, +\partial^m, -\partial^m\}$.

Definition 12. *A proof P is a sequence* $P(1)\dots P(n)$ *of tagged literals satisfying the proof conditions given in Definitions 15, 16, 17 and 18. Each* $P(i)$, $1 \leq i \leq n$ *is called a line of the proof. Given a proof P,* $P(1..n)$ *denotes the first n lines of the proof.*

Definition 13. *A rule r is applicable at index i in a proof P at line* $P(n+1)$ *iff*[6]

1. $\forall a \in A(r)$:
   (a) *if* $a \in TLit$, *then* $a \in F$, *and*
   (b) i. *if* $a = Ol^t$, *then* $+\partial l^t \in P(1..n)$,
       ii. *if* $a = \neg Ol^t$, *then* $-\partial l^t \in P(1..n)$; *and*
2. $\forall c_j \in C(r), 1 \leq j \leq i$:
   (a) *if* $mode(c_j) = p$, *then* $c_j \notin F$ *or* $\sim c_j \in F$,
   (b) *if* $mode(c_j) = a$, *then* $\forall t$, $start(c_j) \leq t \leq end(c_j)$, $c_j^t \notin F$ *or* $\sim c_j^t \in F$,
   (c) *if* $mode(c_j) = m$, *then* $\exists t$, $start(c_j) \leq t \leq end(c_j)$, $c_j^t \notin F$ *or* $\sim c_j^t \in F$.

Definition 14. *A rule r is discarded at index i in a proof P at line* $P(n+1)$ *iff*

1. $\exists a \in A(r)$:
   (a) *if* $a \in TLit$, *then* $a \in F$; *or*
       i. *if* $a = Ol^t$, *then* $-\partial l^t \in P(1..n)$,
       ii. *if* $a = \neg Ol^t$, *then* $+\partial l^t \in P(1..n)$; *or*
2. $\forall c_j \in C(r), 1 \leq j \leq i$
   (a) *if* $mode(c_j) = p$, *then* $c_j \in F$,
   (b) *if* $mode(c_j) = a$, *then* $\forall t$, $start(c_j) \leq t \leq end(c_j)$, $c_j^t \in F$,
   (c) *if* $mode(c_j) = m$, *then* $\exists t$, $start(c_j) \leq t \leq end(c_j)$, $c_j^t \in F$.

In the proof conditions below we will simply use applicable/discarded at index i, instead of applicable/discarded at index i in the proof P at line $P(n+1)$.

All proof tags presented in the paper will be defined according the principle of strong negation [2]. According to it, the pair of tags $+\#$ and $-\#$ are the strong negation of each other, where the strong negation is a function replacing/exchanging: $\forall$

[6] In the following, if ${}^{x_1}c_1^{t_{c_1}} \otimes^{x_2}_{t'_{c_1}} \cdots \otimes^{x_j}_{t'_{c_{j-1}}} c_j^{t_{c_j}} \otimes^{x_{j+1}}_{t'_j} \cdots \otimes_{t'_n} \bot$ is an $\otimes$-chain of length $n+1$, $mode(c_j) = x_j$, $start(c_j) = t_{c_j}$, and $end(c_j) = t'_{c_j}$.

and $\exists$, conjunctions and disjunctions, and 'applicable' and 'discarded'. For space reasons, we provide the definition of both the positive and negative proof tags for punctual obligation (i.e., $+\partial^p$ and $-\partial^p$), and only the positive definition of the proof tags for achievement and maintenance obligations; the corresponding negative proof tags can be derived using the above mentioned principle.

Definition 15 (Proof Conditions for $\pm\partial^p$).

If $P(n+1) = +\partial^p p^t$ then
(1) $\exists r \in R^p_{\Rightarrow}[p^t, i]$ r is applicable at index i and
(2) $\forall s \in R[\sim p^t, j]$, either
(2.1) s is discarded at index j or
(2.2) $\exists w \in R[p^t, k]$ such that w is applicable at k and $w \succ s$.

If $P(n+1) = -\partial^p p^t$ then
(1) $\forall r \in R^p_{\Rightarrow}[p^t, i]$ either r is discarded at i, or
(2) $\exists s \in R[\sim p^t, j]$ such that
(2.1) r is applicable at index j and
(2.2) $\forall w \in R[p^t, k]$ either w is discarded at k or $s \not\succ w$.

The proof conditions above are essentially a simple combination of the condition for $\otimes$ given in [12] and those for punctual obligation of [16]. To prove $+\partial^p a^t$, there must be a rule for a^t such that all the antecedents have to be provable, and for all elements preceding a^t in the head, we have to ensure that a violation occurred. This means that we have to examine the mode of the conclusions at indexes lower that the index of a^t, and then for a punctual obligation we have to see that the content of the obligation did not happen at t. We have two cases: the first is that we do not have a^t in the set of facts, and second we have the opposite, i.e., we have $\sim a^t$. For an achievement obligation we have to check that for all instants in the interval the same condition as that for a punctual obligation is satisfied, while for a maintenance obligation, a violation occurs when the condition holds for at least one instant of time in the interval. Condition (2.1) and (2.2) are the usual conditions of Defeasible Logic, that is: we have to verify that rules for the opposite either do not fire (2.1), they are not applicable, or (2.2) they are defeated by applicable rules for the conclusion we want to prove.

Definition 16 (Proof Conditions for $\pm\partial^a$).

If $P(n+1) = +\partial^a p^t$ then
(1) $\exists r \in R^a_{\Rightarrow}[p^t, i]$ r is applicable at index i and
(2) $\forall s \in R[\sim p^t, j]$, either
(2.1) s is discarded at index j or
(2.2) $\exists w \in R[p^t, k]$ such that w is applicable at k and $w \succ s$; or
(3) $\exists x \in R^a_{\Rightarrow}[p^{t'}, i]$, $t' < t$, $end(p^{t'}) \geq t$ and
(3.1) x is applicable at index i, and
(3.2) $\forall y \in R[\sim p^{t''}, j]$, $t' \leq t'' < t$ either
(3.2.1) y is discarded at j or
(3.2.3) $\exists z \in R[p^{t''}, k]$, z is applicable at k and $z \succ y$; and
(3.3) $\forall t''', t'' < t''' \leq t$, $p^{t'''} \notin F$.

The conditions for $+\partial^a p^t$ are similar to those for punctual obligations. The differences are that we have to consider persistence, clause (3). This means that we could have derived the obligation in the past, let us say at time t', and the obligation has not been terminated since them. We have two ways to terminate it: there is a rule for the opposite that is applicable between t and t' (3.2) see [16], or the obligation has been already fulfilled (3.3).

Definition 17 (Proof Conditions for $\pm\partial^m$).

If $P(n+1) = +\partial^m p^t$ then
(1) $\exists r \in R^m_{\Rightarrow}[p^t, i]$ r is applicable at index i and
(2) $\forall s \in R[\sim p^t, j]$, either
  (2.1) s is discarded at index j or
  (2.2) $\exists w \in R[p^t, k]$ such that w is applicable at k and $w \succ s$; or
(3) $\exists x \in R^m_{\Rightarrow}[p^{t'}, i]$, $t' < t$, $end(p^{t'}) \geq t$ and
  (3.1) x is applicable at index i, and
  (3.2) $\forall y \in R[\sim p^{t''}, j]$, $t' \leq t'' < t$ either
    (3.2.1) y is discarded at j or
    (3.2.3) $\exists z \in R[p^{t''}, k]$, z is applicable at k and $z \succ y$.

The conditions for maintenance obligations are the same as those for achievement obligation with the difference that fulfilling the obligation does not terminate it.

Definition 18 (Proof Condition for $\pm\partial$). *If $P(n+1) = +\partial p^t$, then either $+\partial^p p^t \in P(1..n)$, or $+\partial^a p^t \in P(1..n)$, or $+\partial^{a^p r} p^t \in P(1..n)$, or $+\partial^m p^t \in P(1..n)$.*

If $P(n+1) = -\partial p^t$, then $-\partial^p p^t \in P(1..n)$, and $-\partial^a p^t \in P(1..n)$, and $+\partial^{a^p r} p^t \in P(1..n)$, and $+\partial^m p^t \in P(1..n)$.

Definition 19. *Given a theory D, the* universe *of D (U^D) is the set of all the atoms occurring in D. The* extension *E^D of D is a structure (∂^+, ∂^-), where, for $X \in \{p, a, m\}$, $\partial^+_D = \{l^t : D \vdash +\partial^X l^t\}$ and $\partial^-_D = \{l^t : D \vdash -\partial^X l^t\}$.*

Example 7. Consider the following theory:

$$
\begin{aligned}
F = \{&Invoice^t, \neg Pay^t, \neg Pay^{t+1}, PayInterest^{t+2}, Defective^t\}\\
R = \{&r_1 : Invoice^t \Rightarrow^a Pay^t \otimes_{t+1} \bot\\
&r_2 : Invoice^t, OPay^{t+1}, \neg Pay^{t+1} \Rightarrow^a PayInterest^{t+2} \otimes_{t+3} \bot,\\
&r_3 : Defective^t \rightsquigarrow \neg Pay^t\}\\
\succ = \{&r_1 \succ r_3\}
\end{aligned}
$$

The first two norms basically describe the same situation of Example 6: the only difference is that here we have not yet applied any introduction rule for $\otimes$. r_3 states that, if the delivered good is defective, the customer is allowed not to pay. The facts trigger r_1, thus we derive the obligation to pay by $t+1$ (starting from t): also r_3 is triggered but is weaker than r_1. The obligation to pay is however not fulfilled by F. Since $\neg Pay^t \in F$, we obtain $OPay^{t+1}$ from r_1, which contributes to triggers r_2, thus obtaining the obligation to pay the interest by $t+3$ (starting from $t+2$). Since the obligation to pay by $t+1$ is not fulfilled, the extension of the theory D contains $\bot$: r_1 was not complied with.

5 Checking Compliance

If we work on the idea that a set of facts may fulfill a set of norms even when some of these norms are violated (but such violations are always compensated), then the following definition of compliance does not suffice:

Definition 20 (Theory compliance). *A Defeasible Theory D is compliant iff $\bot \notin \partial_D^+$.*

Definition 20 is very simple and exploits the basic properties of any temporalized obligations: since all $\otimes$-chains have $\bot$ as their last element, they have an ultimate deadline beyond which we derive $\bot$: this amounts to saying that after that deadline we state that it is impossible to compensate. Since the proof conditions for our logic establish that an obligation in an $\otimes$-chain is derived only if the previous obligations in that chain are violated, if we have $\bot$ in the positive extension of a theory, this means that there is at least one obligation whose violation cannot be compensated. For instance, if we consider Example 7, according to Definition 20 the theory D is not compliant because the theory extension contains $\bot$. However, such a theory should be considered compliant, since norm r_2, which provides a compensation for the violation of r_1, is indeed fulfilled.

Normalisation Process The inference rules $(\otimes I_p)$, $(\otimes I_m)$, and $(\otimes I_a)$ provide a method for representing the norms in a format that can be used to check the compliance of a theory. In fact, they allow for making explicit the hidden reparative relation between obligations. Once applied, the redundant rules can be removed. For instance, in Example 7 above, we could apply $(\otimes I_a)$ to r_1 and r_2 and obtain the new rule

$$r_3 : \mathit{Invoice}^t \Rightarrow^a \mathit{Pay}^t \otimes^a_{t+1} \mathit{PayInterest}^{t+2} \otimes_{t+3} \bot$$

Once r_3 is obtained, since r_2 is subsumed by r_3, then r_2 is deontically redundant and can be removed from the theory.

Formally, this process is called normalisation of a theory. Before presenting the process, some auxiliary notions are needed: (a) Definition 21 identifies all the instances of inference rules we can obtain from a theory; (b) since such instances allow to introduce new norms, we should establish when these norms can inherit the same strength qualifications (via $\succ$) of previous norms; we should also remove redundant norms and norm priorities (Definitions 22 and 23); (c) Definition 24 introduces the deductive closure of a theory under the inference conditions for $\otimes$.

Definition 21. *Let $D = (F, R \succ)$ be any defeasible theory. Any instance I of the inference rules $(\otimes I_p)$, $(\otimes I_m)$, and $(\otimes I_a)$ is* based on *D if each of the premises r_i and r_j of I is either (a) in R (in which case, the instance is* rooted*), or (b) is the conclusion of another instance of the inference rules $(\otimes I_p)$, $(\otimes I_m)$, and $(\otimes I_a)$ based on D.*

The instances of the inference rules $(\otimes I_p)$, $(\otimes I_m)$, and $(\otimes I_a)$ based on D are also called D-$\otimes$-instances.

Definition 22. *Let $D = (F, R \succ)$ be any defeasible theory. The superiority relation $\succ^\infty = \cup_{i=1}^\infty \succ_i$ is recursively defined as follows:*

- *$\succ_0 = \succ \cup \{(j,k) \mid j$ (or k) is the conclusion of a rooted D-$\otimes$-instance such that $k \in R$ (or $j \in R$) and, for any $i \in R$, $(i,k) \in \succ$ (or $(j,i) \in \succ$) $\}$;*

– $\succ_{i+1} = \succ_i \cup \{(j,k) |$ *j (or k) is the conclusion of a D-$\otimes$-instance such that* $(i,k) \in \succ_i$ *(or* $(j,i) \in \succ_i$*)* $\}$.

The relation $\succ^\infty$ *is called the D*-saturation of $\succ$.

Definition 23. *Let $D = (F, R \succ)$ be any defeasible theory. Let $\mathscr{S}$ be an operation over D defined as follows: if $\Pi = \{r | r \in R, \exists r' \in R : r'$ subsumes $r\}$, then*

$$\mathscr{S}(D) = \begin{cases} D' & \text{where } D' = (F, R', \succ') \text{ such that} \\ & R' = R - \Pi \text{ and} \\ & \succ' = \succ^\infty - \{(x,y) \in \succ \mid \text{either } x \in \Pi \text{ or } y \in \Pi\} \\ D & \text{otherwise} \end{cases} \tag{10}$$

Definition 24. *If $D = (F, R \succ)$ is any defeasible theory, let $\vdash_\otimes$ be the consequence relation defined by the inference rules $(\otimes I_p)$, $(\otimes I_m)$, and $(\otimes I_a)$. The* closure $(D, \vdash_\otimes)$ *of D under $\vdash_i$ is a theory $D' = (F, R', \succ')$ where (a) R' is the smallest set containing all elements of R and the conclusions of all D-$\otimes$-instances; (b) $\succ'$ is the D*-saturation of $\succ$.

Definition 25 (Theory normalisation). *The* normalisation D^∞ *of a theory D is a theory recursively obtained as follows: (a) $D_0 = D$, (b) $D_{i+1} = \mathscr{S}(D_i, \vdash_\otimes)$.*

The inference rules and the rule removal via subsumption must be done several times in the appropriate order. The normalised theory is the fixed-point of the above constructions. At each step of the the procedure we have to first apply the inference rules for $\otimes$ and then the subsumption: suppose we have a theory containing the following three norms

$r_1 : f^{t_f} \Rightarrow^p a^{t_a} \otimes^p_{t_a} g^{t_g} \otimes_{t_g} \bot$ $\qquad$ $r_2 : e^{t_e} \Rightarrow^p a^{t_a} \otimes^p_{t_a} b^{t_b} \otimes^p_{t_b} c^{t_c} \otimes^p_{t_c} d^{t_d} \otimes_{t_d} \bot$

$r_3 : e^{t_e}, \neg a^{t_a}, \neg b^{t_b} \Rightarrow^p c^{t_c} \otimes_{t_c} \bot$

The normalisation process would consist here in a single cycle leading to apply (i) $(\otimes Ip)$ to r_1 and r_3, thus producing $r_4 : e^{t_e}, f^{t_f}, \neg b^{t_b} \Rightarrow^p a^{t_a} \otimes^p_{t_a} c^{t_c} \otimes_{t_c} \bot$; (ii) subsumption and remove r_3. Notice that also r_2 subsumes r_3. However, if we apply subsumption first on this basis we have to delete r_3 and r_4 would be no longer derivable from r_1 and r_3 alone.

After a theory is normalised, Definition 20 can be safely applied, as all redundant rules are removed and all hidden reparative connections between obligations are made explicit.

Finally, notice that (i) the structure of the inference rules $(\otimes I_p)$, $(\otimes I_m)$, and $(\otimes I_a)$ states that one premise in all instances is subsumed by the conclusion and so is removed at the end of each step of the process; (ii) any defeasible theory contains only finitely many rules and each rule has finitely many elements; also the operation on which the construction is defined is monotonic [14].

If a superiority relation $\succ$ is consistent iff $(x,y),(y,x) \notin \succ$, then reason (i) above supports the following result:

Proposition 1. *For any defeasible theory D, the normalisation $D^\infty = (F, R, \succ^\infty)$ is such that $\succ^\infty$ is consistent.*

Also, so by standard set theory results, reason (ii) above supports the following:

Proposition 2. *The normalisation D^∞ of any defeasible theory D exists and is unique.*

6 Summary and Related Work

This paper extends the logic of violation proposed by [14] with time. This extension introduces a temporal dimension to the language saying when a norm produces its normative effects, or in other terms when the obligation (or, in general the normative position) corresponding to the normative effect of the norm is in force. An immediate consequence of the extended language is that it is possible to investigate the 'lifecycle' of obligations, and more precisely if there are deadlines to comply with an obligation. The extension is done to properly deal with the concept of legal compliance. To do this we argue that we have to handle different types of temporalised legal obligations and devise a normalisation procedure for making hidden conditions and reparative chains explicit. One open research issue is to investigate the complexity of this procedure, which requires, several times and in the appropriate order, to apply the inference rules for $\otimes$ and to remove redundant norms.

The literature on norm compliance is MAS is large (see, e.g., [5,9,20,10,1,11,17,3,19]). However, to the best of our knowledge no work in the field has so far attempted to model *legal* compliance pertaining to realistic systems where complex norm-enforcement mechanisms such as reparative chains are combined with a rich ontology of obligations as the one described here. In the literature on deontic logic, besides a few exceptions like [6], the research has mostly devoted extensive, but separate, efforts to the role of time for dealing with CTDs (since the seminal [25]) and on logical systems for modeling the concept deontic preference and CTDs (for an overview, [23]). This paper combines the two perspectives: in this sense, it also inherits from [14] the advantage of avoiding the most well-known CTD paradoxes. In this sense, [6] shares with our paper the same general view, but time is captured there at the semantic level and the language does not explicitly handle timestamps.

Combination of time and norms are not novel, as many combinations of temporal (or tense) logic and deontic logic have been investigated. However, temporal logic cannot handle specific times (or timestamps). Typically these logics can express the temporal relationships between events (represented by propositions), or the relationships between states. A possible solution to obviate this is to consider hybrid logic using nominals to capture nominals [22]. A nominal represents a proposition true only in one possible worlds. A temporal nominal represents a particular instant of time. In most temporal logic it is possible to model branching of time, and the meaning of nominals is not clear in this kind of situations (is the world corresponding to a nominal the same in all the branches, or we have different copies of the same instant of time?). On the other hand timestamps (and events) have been used in the Event Calculus. Event Calculus has been used to model the interaction between norms and time (see, e.g., [21]). However, Event Calculus is a dialect of first-order logic and Herrestad [18] has shown that these types of logic are not suitable to model normative reasoning in presence of violations.

References

1. M. Alberti, M. Gavanelli, E. Lamma, F. Chesani, P. Mello, and P. Torroni. Compliance verification of agent interaction: a logic-based software tool. *Applied Artificial Intelligence*, 20(2-4):133–157, 2006.

2. G. Antoniou, D. Billington, G. Governatori, and M. Maher. A flexible framework for defeasible logics. In *Proc. AAAI-2000*. AAAI Press, 2000.
3. G. Boella, J. Broersen, and L. van der Torre. Reasoning about constitutive norms, counts-as conditionals, institutions, deadlines and violations. In *PRIMA*. Springer, 2008.
4. G. Boella and L. van der Torre. Fulfilling or violating obligations in multiagent systems. In *Procs. IAT04*, 2004.
5. E. Bou, M. López-Sánchez, and J. A. Rodríguez-Aguilar. Adaptation of autonomic electronic institutions through norms and institutional agents. In *Proc. ESAW'06*. Springer, 2006.
6. J. Broersen and L. van der Torre. Conditional norms and dyadic obligations in time. In *Proc. ECAI 2008*. IOS Press, 2008.
7. J. Carmo and A. Jones. Deontic logic and contrary to duties. In D. Gabbay and F. Guenther, editors, *Handbook of Philosophical Logic, 2nd Edition*. Kluwer, 2002.
8. M. Dastani, D. Grossi, J.-J. C. Meyer, and N. Tinnemeier. Normative multi-agent programs and their logics. In R. Bordini, M. Dastani, J. Dix, and A. E. Fallah-Seghrouchni, editors, *Programming Multi-Agent Systems*, number 08361 in Dagstuhl Seminar Proceedings, Dagstuhl, Germany, 2008. Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik, Germany.
9. M. Esteva, B. Rosell, J. A. Rodríguez-Aguilar, and J. L. Arcos. Ameli: An agent-based middleware for electronic institutions. In *Proc. AAMAS 2004*. ACM, 2004.
10. R. A. Flores and B. Chaib-draa. Modelling flexible social commitments and their enforcement. In *Proc. Engineering Societies in the Agents World V*. Springer, 2004.
11. D. Gaertner, A. Garcia-Camino, P. Noriega, J.-A. Rodriguez-Aguilar, and W. Vasconcelos. Distributed norm management in regulated multiagent systems. In *Proc. AAMAS '07*. ACM, 2007.
12. G. Governatori. Representing business contracts in RuleML. *International Journal of Cooperative Information Systems*, 14(2-3):181–216, 2005.
13. G. Governatori, J. Hulstijn, R. Riveret, and A. Rotolo. Characterising deadlines in temporal modal defeasible logic. In *Proc. Australian AI 2007*, 2007.
14. G. Governatori and A. Rotolo. Logic of violations: A Gentzen system for reasoning with contrary-to-duty obligations. *Australasian Journal of Logic*, 4:193–215, 2006.
15. G. Governatori and A. Rotolo. An algorithm for business process compliance. In G. Sartor, editor, *Jurix 2008*, pages 186–191. IOS Press, 2008.
16. G. Governatori, A. Rotolo, and G. Sartor. Temporalised normative positions in defeasible logic. In *10th International Conference on Artificial Intelligence and Law (ICAIL05)*, pages 25–34, 2005.
17. D. Grossi, H. Aldewereld, and F. Dignum. Ubi lex, ibi poena: Designing norm enforcement in e-institutions. In *In Coordination, Organizations, Institutions, and Norms in Multi-Agent Systems II*. Springer, 2006.
18. H. Herrestad. Norms and formalization. In *ICAIL*, pages 175–184, 1991.
19. J. F. Hübner, O. Boissier, and R. Bordini. From organisation specification to normative programming in multi-agent organisations. In *CLIMA XI*, 2010.
20. F. López y López, M. Luck, and M. d'Inverno. Constraining autonomy through norms. In *Proc. AAMAS '02*. ACM, 2002.
21. R. H. Marín and G. Sartor. Time and norms: a formalisation in the event-calculus. In *ICAIL*, pages 90–99, 1999.
22. C. Smith, A. Rotolo, and G. Sartor. Temporal reasoning and mas. In *SNAMAS 2010*, 2010.
23. J. Van Benthem, D. Grossi, and F. Liu. Deontics = betterness + priority. In *Proc. DEON'10*. Springer, 2010.
24. L. van der Torre, G. Boella, and H. Verhagen, editors. *Normative Multi-agent Systems*, Special Issue of *JAAMAS*, vol. 17(1), 2008.
25. J. van Eck. A system of temporally relative modal and deontic predicate logic and its philosophical applications. *Logique et Analyse*, 25:339–381, 1982.