

# Burdens of Proof in Monological Argumentation

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**Abstract:** We shall argue that burdens of proof are relevant also to monological reasoning, i.e., for deriving the conclusions of a knowledge-base allowing for conflicting arguments. Reasoning with burdens of proof can provide a useful extension of current argument-based non-monotonic logics, at least a different perspective on them. Firstly we shall provide an objective characterisation of burdens of proof, assuming that burdens concerns rule antecedents (literals in the body of rules), rather than agents. Secondly, we shall analyse the conditions for a burden to be satisfied, by considering credulous or skeptical derivability of the concerned antecedent or of its complement. Finally, we shall develop a method for developing inferences out of a knowledge base merging rules and proof burdens in the framework of defeasible logic.

## Introduction

In research on legal argumentation two directions can be distinguished: (a) *monological argumentation*, as a method for deriving warranted conclusion from a knowledge base containing defeasible premises, and (b) *dialogical argumentation*, as an interactive process for taking shared commitments. Both approaches address arguments and their interaction (what conclusions are supported by arguments attacking or supporting one-another), but they are different in various regards. In the first approach the argument set under consideration includes all arguments constructible from the given knowledge base (the argumentation framework), the second focuses on arguments that were, as a matter of fact, produced by the parties in the dialogue. In the first approach time is irrelevant (unless it involves a modification of the knowledge base), in the second it matters, since the sequence in which arguments are presented determines their relevance to the dialogue, according to the dialectical protocol governing the argument exchange. The first approach is basically agent-neutral (what matters is what premises are in the knowledge base, regardless of who originally provided them), the second instead is agent-based, since the dialectical function and relevance of an arguments depends on which party proposed it, and on the argumentation burdens that bear upon that party.

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Both approaches have attracted a lot of attention in AI and in particular in AI & Law. Research on monological argumentation has led to various argument-based logics for defeasible reasoning. Research on dialogical argumentation has led to various protocol-based dialectical models of argumentation. We believe that these two lines of research are complementary and should indeed be integrated into a comprehensive model. Here however, we want to focus on a limited subject, namely, *burden of proofs*. We want to argue that, contrary to what has been recently claimed ([5]), there is a way of understanding burdens of proof that makes them relevant also relevant to monological argumentation. By including them in a logic for monological argumentation we can obtain a useful extension of current nonmonotonic logics, or at least a different perspective on them. For this purpose the paper will develop in the following way.

We shall provide an objective characterisation of burdens of proof, as opposed to the subjective characterisation that is usually provided: we view burdens as properties of propositions, or more exactly of literals in rule antecedents, rather than as relationships between agents and propositions. Rather than expressing a burden with a proposition like “agent  $x$  has the burden of proving proposition  $y$ ”, we shall say “there is the burden of proving literal  $w$  in rule  $z$ ”. Burden so understood specify conditions for deriving warranted conclusions out of a knowledge base of rules: a rule can fire, only when the burdens concerning each of its antecedents (each literal in the rule’s body) are satisfied. Burdens themselves can be specified through rules belonging to the same knowledge base: reasoning with burdens and reasoning about burdens are integrated in the same argumentative process.

Following the terminology of [8], we distinguish two burdens of proof, namely, *burden of production* and *burden of persuasion*. However, while in [8] the term “burden of production” was given a procedural meaning, i.e., as indicating which party has to provide evidence for a claim at different points in a proceeding, here we understand it in an inferential way, i.e., as the need for a rule-antecedent to be supported by a credulous argument. This is not meant to exclude the procedural notion, but to provide a different perspective, which needs to be integrated with the procedural one. The ideas we express by speaking of burdens can also be conveyed as a specification of proof-standards, understood as requirements to be met for a proposition to be taken as established, in the context a certain inference (see [5]). However, proof standards can also be understood in different ways, in particular as referring to conditions for argument defeat ([8]). Given the still uncertain articulation of these notions, we maintain here the burden terminology and leave to future research the integration of the notions here provided with further concepts, an integration that may involve a different terminology.

## 1. Rules and burdens

In this section we shall introduce the basic ideas underlying the representation model and inference mechanism here developed. Let us assume that we have a defeasible logic (an argumentation framework) that allows us distinguish two types of conclusions: skeptical conclusions (warranted by arguments in the knowledge base) and credulous conclusions (supported by an argument which is not attacked by any skeptical conclusion, though it may be attacked by a credulous one). Other terminologies could be used to express this difference (such as the distinction between justified and defensible conclusions used in

[6]), and various semantics can be used for it (in particular, following the approach of [4]). Let us write  $KB \vdash^s p$  to mean that  $p$  is a *skeptical conclusion* of the knowledge base  $KB$ ,  $KB \vdash^c p$  to mean that  $p$  is a *credulous conclusion* of it, and let us say that  $p$  is a *merely credulous* (credulous and not skeptical) if  $KB \vdash^c p$  and  $KB \not\vdash^s p$ . Let us also assume that a rule has the usual syntax:

$$r: p_1 \wedge \dots \wedge p_n \Rightarrow q$$

where the rule-name  $r$  is a literal, the antecedents  $p_1 \wedge \dots \wedge p_n$  as well as the consequent  $q$  are literals and  $\Rightarrow$  denotes a defeasible conditional. Here is a simple legal example:

$$r: \text{tort} \wedge \neg \text{justification} \Rightarrow \text{liability}$$

meaning that if there is a tort and there is no justification, then there is liability. This rule does not contain all the information necessary for its application. It must be supplemented with indications concerning proof requirements: the rule will fire only when, for every antecedent  $p_i$ ,  $p_i$ 's proof requirements are satisfied. To determine what proof-requirements exist with regard to a literal  $p$  in a rule  $r$ , we need consider whether proof-burdens are on  $p$  or on its complement  $\bar{p}$ . We distinguish two burdens, the burden of production and the burden of persuasion and write respectively  $BPr(r, p)$  to mean that wrt rule  $r$  there is burden of production on  $p$  and  $BPe(r, p)$  to mean that wrt to rule  $r$  there is burden of persuasion on  $p$ .

The *burden of production* provides guidance for situations where no credulous arguments against  $p$  are available. If  $BPr(r, \bar{p})$ , then to satisfy antecedent  $p$  in rule  $r$  it will be sufficient that that  $KB \not\vdash^c \bar{p}$ , regardless of requirements of persuasion. On the contrary, if  $BPr(r, p)$ , this is not sufficient. To satisfy  $p$  supporting arguments are necessary: it must be the case that either  $KB \vdash^c p$  or  $KB \vdash^p p$  according to the applicable requirement of persuasion. The *burden of persuasion* provides guidance for situations where  $p$  only is a merely credulous conclusion. If  $BPe(\bar{p}, r)$ , then to satisfy  $p$  it is sufficient that  $KB \vdash^c p$  (even though  $KB \not\vdash^s p$ ). On the contrary if  $BPe(p, r)$ , then  $KB \vdash^c p$  is not sufficient. In this case,  $KB \vdash^s p$  will provide sufficient support for satisfying  $p$ .

It is not necessary that the two burdens go together: in particular, it may happen that  $BPr(r, \bar{p})$  and the  $BPe(r, p)$ . They correspond indeed to different kinds of implicit inferences: by allocating the burden of production on  $\bar{p}$ , we commit to presuming  $p$  (for the purpose of applying a  $r$ ) in the absence of relevant information to the contrary; by allocating the burden of persuasion on  $\bar{p}$ , we commit to presuming  $p$  (for the purpose of applying  $r$ ) in the presence of doubtful reasons supporting  $p$ . This explain while the first may be applicable when the second is not: it may be reasonable to presume (e.g., for the purpose of allocating criminal liability) that there are no justifications for a crime when no reasons have been provided to that effect (if there were justifications we would have known something about that), while it may be unreasonable (or too harsh on the accused) to presume that there are no justifications in the presence of doubts. Note that this treatment of presumptions is different from that developed in [7] since here presumptions are local to a rule. Given an antecedent  $p$  in a rule  $r$ , by allocating a burden of production on  $\bar{p}$ , we are committed to view  $p$  as satisfied whenever there is no argument for  $\bar{p}$ , but we are not committed to treat in the same way the occurrences of  $p$  in other rules. Thus we may handle contexts where the law presumes a certain fact only for a certain purpose (for the application of a certain rule) and not for other purposes (for using other rules).

By assigning both burden of production and a burden of persuasion either to  $p$  or to  $\bar{p}$ , we get the following combinations:

1.  $BPr(r, p) \wedge BPe(r, p)$ : the proof-requirements of  $p$  in rule  $r$  are satisfied iff  $KB \vdash^s p$ . In fact, given that  $BPr(r, p)$  support for  $p$  is required (it is not sufficient that  $KB \not\vdash^c \bar{p}$ ), and since  $BPe(r, p)$ ,  $p$  must be skeptically supported.
2.  $BPr(r, p) \wedge BPe(r, \bar{p})$ : the proof-requirements of  $p$  in rule  $r$  are satisfied iff  $KB \vdash^c p$  and  $KB \not\vdash^s \bar{p}$ . Given that  $KB \vdash^c p$  entails  $KB \not\vdash^s \bar{p}$ , it is sufficient that  $p$  is credulously supported.
3.  $BPr(r, \bar{p}) \wedge BPe(r, p)$ : the proof-requirements of  $p$  in rule  $r$  are satisfied iff one of the following holds: (3a)  $KB \not\vdash^c \bar{p}$  or (3b)  $KB \vdash^s p$ . Thus, there are two alternative sufficient conditions for satisfying  $p$ : according to (3a)  $BPr(r, \bar{p})$ ,  $p$  is satisfied if  $KB \not\vdash^c \bar{p}$  while according to (3b)  $BPe(r, p)$ ,  $p$  is satisfied if  $KB \vdash^s p$ .
4.  $BPr(r, \bar{p}) \wedge BPe(r, \bar{p})$ : the proof-requirements of  $p$  in rule  $r$  are satisfied iff one of the following holds: (4a)  $KB \not\vdash^c \bar{p}$  or (4b)  $KB \vdash^c p$ . Thus, there are two alternative sufficient conditions for satisfying  $p$ : according to (4a)  $BPr(r, \bar{p})$ ,  $p$  is satisfied if  $KB \not\vdash^c \bar{p}$  while according to (4b)  $BPe(r, \bar{p})$ ,  $p$  is satisfied if  $KB \vdash^c p$ .

## 2. An example

With the resources of the distinction between the two kinds of burden we can address the distinctive role that different operative fact have in the antecedent of legal norms. Let us consider the following example, which shows the different way in which liability is dealt with in criminal law and in civil law.

$$\begin{aligned}
KB_0 = \{ & r_1: \text{tort} \wedge \neg\text{justification} \rightarrow \text{civilLawLiability}, \\
& r_2: \text{crime} \wedge \neg\text{justification} \rightarrow \text{criminalLawLiability}, \\
& b_{1a0}: BPr(r_1, \text{tort}), b_{1a1}: BPe(r_1, \text{tort}), \\
& b_{1b0}: BPr(r_1, \text{justification}), b_{1b1}: BPe(r_1, \text{justification}), \\
& b_{2a0}: BPr(r_2, \text{crime}), b_{2a1}: BPe(r_2, \text{crime}), \\
& b_{2b0}: BPr(r_2, \text{justification}), b_{2b1}: BPe(r_2, \neg\text{justification}) \}
\end{aligned}$$

In labelling statements about burdens, we follow the convention of indicating first the number of the rule, then the position of the element at issue in the rule, then 0 for burden of production and 1 for burden of persuasion.

Assume that the  $KB_0$  is extended with the information that indeed there was a crime, and that there was a tort:

$$KB_1 = KB_0 + \{r_3: \text{crime}, r_4: \text{tort}\}$$

At this point we have skeptical arguments for both *tort* and *crime*, and we have no argument neither for *justification* nor for  $\neg\text{justification}$ . Let us take into consideration the two norms  $r_1$  and  $r_2$ . It is easy to see that the antecedents elements of both are satisfied. Let us start with  $r_1$ . The proof requirements for the first element of  $r_1$  namely *tort*, are given by  $BPr(r_1, \text{tort}) \wedge BPe(r_1, \text{tort})$  (case 1 in the enumeration at the end of previous section, i.e.,  $BPr(r, p) \wedge BPe(r, p)$ ). In  $KB_1$  these requirements are met since  $KB_1 \vdash^s \text{tort}$ . The proof requirements for the second element of  $r_1$ , namely,  $\neg\text{justification}$ , are given by  $BPr(r_1, \text{justification}) \wedge BPe(r_1, \text{justification})$ : both burdens fall upon the complement of  $\neg\text{justification}$ , i.e., upon *justification* (case 4, i.e.,  $BPr(r, \bar{p}) \wedge BPe(r, \bar{p})$ ). In  $KB_1$  the proof requirements for  $\neg\text{justification}$  are met since  $KB_1 \not\vdash^c \text{justification}$  (alternative 4a).

Let us now consider  $r_2$ . For the first element, namely *crime* the situation is the same as for *tort*. For the second element, namely,  $\neg$ *justification*, the proof requirements are different: since incriminating norms cannot be applied in case of doubt on their preconditions, there is burden of production upon *justification*, but no burden of persuasion:  $BPr(r_1, justification) \wedge BPe(r_1, \neg justification)$  (case 3, i.e.,  $BPr(r, \bar{p}) \wedge BPe(r, p)$ ). However, this does not make any difference in this context: given that  $BPr(r_1, justification)$ ,  $p$  is satisfied since  $KB_1 \not\vdash^c justification$  (alternative 3a). Thus we can derive in  $KB_1$  both *civilLawLiability* and *criminalLawLiability*.

Let us now assume that we add to  $KB_1$  two conflicting pieces of evidence concerning justification, e.g., the defendant having been threatened is a reason for concluding that he was acting in self-defence, while the fact that he was physically stronger is a reason concluding that this was not the case.

$$KB_2 = KB_1 + \{r_5: threat \rightarrow justification, r_6: stronger \rightarrow \neg justification, \\ r_7: threat, r_8: stronger\}$$

Now we have merely credulous arguments for both *justification* and  $\neg$ *justification*. Since  $KB_1 \vdash^c justification$ , the  $\neg$ *justification* elements in  $r_1$  and  $r_2$  cannot be satisfied by referring to alternatives (4a) and (3a) above (which require that  $KB_1 \not\vdash^c justification$ ). The burdens of persuasion become crucial: while in the civil law rule  $r_1$  we have  $BPe(r_1, justification)$ , in the penal law rule  $r_2$  we have  $BPe(r_2, \neg justification)$ . Consequently,  $\neg$ *justification* is satisfied in the civil law rule  $r_1$  (for which it is sufficient that  $KB_1 \vdash^c \neg justification$ , according to 4b above), while it is not satisfied in the criminal law  $r_2$  (for which it is required that  $KB_1 \vdash^s \neg justification$ , according to 3b).

In conclusion, we can derive (and indeed sceptically derive) *civilLawLiability* while we fail to derive *criminalLawLiability*.

### 3. Arguments about burdens

In this example above assertions about burdens are included in the knowledge-base. However, they may also be derived from the knowledge base through further inferences. Consider the following example:

$$KB = \{r_6: damage \wedge negligence \rightarrow tort, r_7: damage, \\ b_{6a0}: BPr(r_6, damage), b_{6a1}: BPe(r_6, damage), \\ b_{6b0}: BPr(r_6, negligence), b_{6b1}: BPe(r_6, negligence), \\ b_{7a0}: medicalCase \rightarrow BPr(r_6, \neg negligence), \\ b_{7b0}: medicalCase \rightarrow BPe(r_6, \neg negligence), \\ p_1: b_{7a0} \succ b_{6a0}, p_2: b_{7b0} \succ b_{6b0}\}$$

Since there is a medical case we can establish that the burden of production is on  $\neg$ *negligence* (rather than on *negligence*), which allows us to conclude for liability only on the basis of *damage*.

### 4. Modelling Burden of Proof in Defeasible Logic

In this section we illustrate how to formalise the ideas presented in the previous section using Defeasible Logic. Defeasible Logic is an efficient rule-base skeptic non-monotonic

formalism that is able to capture different, and sometimes incompatible facets of non-monotonic reasoning, in particular the framework of [1,3] can be used to define several variants of the logic. Even if we speak of variants, each variant is characterised by different proof conditions, conditions under which (tagged) literals can be derived or in other terms inference rules. However, proof conditions for different variants can co-exist, thus we can define a new variant incorporating various variants. In this paper we concentrate on the ambiguity blocking and ambiguity propagating variants. The ambiguity blocking is the basic variant of defeasible logic [2], while the ambiguity propagating one [3] allows us to define credulous conclusions.

Defeasible Logic has three types of rules, strict rules, defeasible rules and defeaters, for simplicity, we will restrict ourselves just to defeasible rules (this assumption is not a limitation, since it is possible to remove defeaters and strict rules for the computation of the non-monotonic consequences without affecting the result [2]). A rule  $r$  consists of its *antecedent*  $A(r)$  which is a finite (possibly empty) set of literals, and its *consequent* (or *head*)  $C(r)$ , which is a single literal.

A *defeasible theory* is a structure  $D = (F, R, >)$  where  $F$  is a set of facts, represented as literals,  $R$  is a set of rules, and  $>$ , the superiority relation, is a binary relations establishing the relative strength of rule. Thus given two rules, let us say  $r_1$  and  $r_2$ ,  $r_1 > r_2$  means that  $r_1$  is stronger than  $r_2$ , thus  $r_1$  can override the conclusion of  $r_2$ . For a literal  $p$ , the set of rules whose head is  $p$  is denoted by  $R[q]$  and  $\sim p$  denotes the complement of  $p$ , that is  $\neg q$  is  $p = q$  and  $q$  is  $p = \neg q$ .

A *conclusion* of  $D$  is a tagged literal and can have one of the following forms:

1.  $+\partial p$ :  $p$  is defeasibly provable in  $D$  using the ambiguity blocking variant;
2.  $-\partial p$ :  $p$  is defeasibly rejected in  $D$  using the ambiguity blocking variant;
3.  $+\delta p$ :  $p$  is defeasibly provable in  $D$  using the ambiguity propagation variant;
4.  $-\delta p$ :  $p$  is defeasibly rejected in  $D$  using the ambiguity propagation variant;
5.  $+\sigma p$ :  $p$  is supported in  $D$ , i.e., there is a chain of reasoning leading to  $p$ ;
6.  $-\sigma p$ :  $p$  is not supported in  $D$ .

The proof tags determine the strength of a derivation. The proof tags  $+\delta$ ,  $-\delta$ ,  $+\partial$  and  $-\partial$  are for skeptical conclusions, and  $+\sigma$  and  $-\sigma$  capture credulous conclusions.

A proof (or derivation)  $P$  is a finite sequence  $(P(1), \dots, P(n))$  of proof tags, satisfying the proof conditions (corresponding to inference conditions) presented in the rest of this section. The proof conditions, given a derivation  $P(1), \dots, P(n)$ , describe the conditions under which we can extend the derivation to derive  $P(n+1)$ . We use the notation  $P(1..n)$  to denote the sequence of length  $n$  of a derivation  $P$ .

$+\partial$ : If  $P(n+1) = +\partial q$  then either

- (1)  $q \in F$  or
- (2)(2.1)  $\exists r \in R[q] \forall a \in A(r): +\partial a \in P(1..n)$  and
  - (2.2)  $\sim q \notin F$  and
  - (2.3)  $\forall s \in R[\sim q]$  either
    - (2.3.1)  $\exists a \in A(s): -\partial a \in P(1..n)$  or
    - (2.3.2)  $\exists t \in R[q]$  such that  $\forall a \in A(t): +\partial a \in P(1..n)$  and  $t > s$ .

$-\partial$ : If  $P(n+1) = -\partial q$  then

- (1)  $q \notin F$  and
- (2) (2.1)  $\forall r \in R[q] \exists a \in A(r): -\partial a \in P(1..n)$  or

- (2.2)  $\sim q \in F$  or
- (2.3)  $\exists s \in R[\sim q]$  such that
  - (2.3.1)  $\forall a \in A(s): +\partial a \in P(1..n)$  and
  - (2.3.2)  $\forall t \in R[q]$  either  $\exists a \in A(t): -\partial a \in P(1..n)$  or  $t \not> s$ .

The main idea of the conditions for a defeasible proof ( $+\partial$ ) is that there is an applicable rule, i.e., a rule whose all antecedents are already defeasibly provable and for every rule for the opposite conclusion either the rule is discarded, i.e., one of the antecedents is not defeasibly provable, or the rule is defeated by a stronger applicable rule for the conclusion we want to prove. The conditions for  $-\partial$  show that any systematic attempt to defeasibly prove that the conclusion fails. Notice that the above conditions characterise the notion of skeptical conclusion using ambiguity blocking [3].

- $+\delta$ : If  $P(n+1) = +\delta q$  then
  - (1)  $+q \in F$  or
  - (2) (2.1)  $\sim q \notin F$  and
    - (2.2)  $\exists r \in R[q] \forall a \in A(r): +\delta a \in P(1..n)$  and
    - (2.3)  $\forall s \in R[\sim q]$  either
      - (2.3.1)  $\exists a \in A(s): -\sigma a \in P(1..n)$  or
      - (2.3.2)  $\exists t \in R[q]$  such that  $\forall a \in A(t): +\delta a \in P(1..n)$  and  $t > s$ .
- $-\delta$ : If  $P(n+1) = -\delta q$  then
  - (1)  $q \notin F$  and
  - (2) (2.1)  $\sim q \in F$  or
    - (2.2)  $\forall r \in R[q]$  either  $\exists a \in A(r): -\delta a \in P(1..n)$  or
    - (2.3)  $\exists s \in R[\sim q]$  such that
      - (2.3.1)  $\forall a \in A(s): +\sigma a \in P(1..n)$  and
      - (2.3.2)  $\forall t \in R[q]$  either  $\exists a \in A(t): -\delta a \in P(1..n)$  or  $t \not> s$ .

The proof tags  $+\delta$  and  $-\delta$  capture defeasible provability using ambiguity propagation [3]. Their explanation is similar to that of  $+\partial$  and  $-\partial$ . The major difference is that to prove  $p$  this time we make easier to attack it (clause 2.3). Instead of asking that the argument attacking it are justified arguments, i.e., rules whose premises are provable, we just ask for defensible arguments (i.e., credulous arguments), that is rules whose premises are just supported (i.e., there is a valid chain of reasoning leading to it). The definition of support, proof tags  $+\sigma$  and  $-\sigma$  is as follows:

- $+\sigma$ : If  $P(i+1) = +\sigma q$  then either
  - (1)  $q \in F$ ; or
  - (2)  $\exists r \in R[q]$  such that
    - (2.1)  $\forall a \in A(r), +\sigma a \in P(1..i)$ , and
    - (2.2)  $\forall s \in R[\sim q]$  either  $\exists a \in A(s), -\delta a \in P(1..i)$ , or  $(s \not> r)$ .
- $-\sigma$ : If  $P(i+1) = -\sigma q$  then
  - (1)  $q \notin F$ , and
  - (2)  $\forall r \in R[q]$  either
    - (2.1)  $\exists a \in A(r), -\sigma a \in P(1..i)$ ; or
    - (2.2)  $\exists s \in R[\sim q]$  such that  $\forall a \in A(s), +\delta a \in P(1..i)$ , and  $s > r$ .

The proof conditions above are essentially for forward chaining of rules to propagate the ‘support’ for arguments (rules). However, we cannot propagate the support from the

premises to the conclusion in case we have a rule for the contrary unless the rule is not weaker than a rule whose premises are all provable.

For a proof tag  $\#$ ,  $D \vdash \#p$  indicates that there is a derivation of  $\#p$  from  $D$ .

**Proposition 1** [3] *Given a defeasible theory  $D$*

- $D \vdash +\delta p$  entails  $D \vdash +\partial p$ , and  $D \vdash +\partial p$  entails  $D \vdash +\sigma p$ ;
- $D \vdash -\sigma p$  entails  $D \vdash -\partial p$ , and  $D \vdash -\partial p$  entails  $D \vdash -\delta p$ ;

We are now ready to formalise the notions described in abstract in the previous section. We start with the notion of skeptical and credulous provability (non provability).

- $KB \vdash^s p$  is mapped to  $D \vdash +\delta p$  or  $D \vdash +\partial p$ ;
- $KB \not\vdash^s p$  is mapped to  $D \vdash -\delta p$  or  $D \vdash -\partial p$ ;
- $KB \vdash^c p$  is mapped to  $D \vdash +\sigma p$
- $KB \not\vdash^c p$  is mapped to  $D \vdash -\sigma p$

The next step is to associate tagged literals to various types of burdens: Let  $p$  be a literal occurring in the body of a rule  $r$ : then the tagged literals associated to the burden are as follows:

- $BPr(r, p): +\sigma p$ ;
- $BPr(r, \bar{p}): -\sigma \sim p$ ;
- $BPe(r, p): +\partial p$ ;
- $BPe(r, \bar{p}): -\partial \sim p$ .

Based on on this mapping, we can allocate the burden based on the following conditions:

1.  $BPr(r, p)$  and  $BPe(r, p): +\delta p$ . The first burden is satisfied by  $+\sigma p$ , and the second by  $+\partial p$ , but according to Proposition 1 the first is implied by the second. Thus we can just consider it.
2.  $BPr(r, p)$  and  $BPr(r, \bar{p}): +\sigma p$ . Similarly to the previous case, the first burden is satisfied by  $+\sigma p$  and the second by  $-\partial \sim p$ . However,  $+\sigma p$  entails  $-\partial \sim p$ .
3.  $BPr(r, \bar{p})$  and  $BPe(r, p): -\sigma \sim p$  or  $+\partial p$ . In this case the burden can be satisfied by any of the two conditions.
4.  $BPr(r, \bar{p})$  and  $BPe(r, \bar{p}): -\sigma \sim p$ . The second burden is satisfied by  $-\partial \sim p$ , but, by Proposition 1 this is entailed by the condition for the burden of production.

To incorporate reasoning about burdens we extend Defeasible Logic in two respects: first we have to incorporate the burden expressions into the language, and we extend the notion of complement as follow: given a rule  $r$  and literal  $l$ ,  $BPr(r, l)$  and  $BPr(r, \sim l)$  are the complement of each other and so are  $BPe(r, l)$  and  $BPe(r, \sim l)$ . Second we extend the proof theory of Defeasible Logic as follows: every time in the course of a derivation we have to derive a tagged literal, we check if the literal has a burden associated to it. If it has, we determine whether we can prove the burden with the appropriate tag, and then, if the condition is satisfied, we check whether the condition associated with the burden is satisfied.

Formally, the clauses of the proof conditions can be rewritten as follows: Let  $\tau$  be a signed proof tag,  $B(r, \tilde{p})$  be a burden, and  $\lambda(r, \tilde{p})$  be the tagged literal associated with  $B(r, \tilde{p})$ ;

If  $\tau$  is positive:

if  $p \in A(r)$ , then if  $\exists s \in R[B(r, \tilde{p})]$ , then  $\tau B(r, \tilde{p}) \in P(1..n)$  and  $\lambda(r, \tilde{p}) \in P(1..n)$ , otherwise  $\tau p \in P(1..n)$ .

If  $\tau$  is negative

if  $p \in A(r)$ , then if  $\exists s \in R[B(r, \tilde{p})]$ , then  $\tau B(r, \tilde{p}) \in P(1..n)$ , otherwise  $\tau p \in P(1..n)$ .

The first of the above clauses checks whether the burden of  $p$  in the context of rule  $r$  is subject of discussion in a case. For example, suppose that it is possible to argue about the burden of persuasion for  $p$  in rule  $r$ . Accordingly, clause (2.2.1) of the proof condition for  $-\sigma$  is rewritten as

$$\forall a \in A(s): +\delta BPr(r, a) \in P(1..n) \text{ and } +\sigma a \in P(1..n).$$

For a negative proof tags, all we have to check, in case there is a burden, is that the burden requirement is not derivable (with the appropriate strenght).

## 5. Defeasible Logic at Work

In this section we illustrate the Defeasible Logic mechanism with the help of the example in Section 3.

$$\begin{aligned} r_1: & \text{tort}, \neg\text{justification} \Rightarrow \text{civilLawLiability}, \\ r_2: & \text{crime}, \neg\text{justification} \Rightarrow \text{criminalLawLiability}, \\ r_3: & \text{threat} \Rightarrow \text{justification}, \\ r_4: & \text{stronger} \Rightarrow \neg\text{justification}, \\ r_5: & \text{damage} \wedge \text{negligence} \Rightarrow \text{tort}, \\ b_{1a0}: & \Rightarrow BPr(r_1, \text{tort}), & b_{1a1}: & \Rightarrow BPe(r_1, \text{tort}), \\ b_{1b0}: & \Rightarrow BPr(r_1, \text{justification}), & b_{1b1}: & \Rightarrow BPe(r_1, \text{justification}), \\ b_{2b0}: & \Rightarrow BPr(r_2, \text{crime}), & b_{2a1}: & \Rightarrow BPe(r_2, \text{crime}), \\ b_{1b0}: & \Rightarrow BPr(r_2, \text{justification}), & b_{2b1}: & \Rightarrow BPe(r_2, \neg\text{justification}), \\ b_{5a0}: & \Rightarrow BPr(r_5, \text{damage}), & b_{5a1}: & \Rightarrow BPe(r_5, \text{damage}), \\ b_{5b0}: & \Rightarrow BPr(r_5, \text{negligence}), & b_{5b1}: & \Rightarrow BPe(r_5, \text{negligence}), \\ b_{7a0}: & \text{medicalCase} \Rightarrow BPr(r_5, \neg\text{negligence}), \\ b_{7b0}: & \text{medicalCase} \Rightarrow BPe(r_5, \neg\text{negligence}), \\ p_1: & b_{7a0} > b_{6a0}, & p_2: & b_{7b0} > b_{6b0}, & f_1: & \text{damage}, & f_2: & \text{medicalCase}. \end{aligned}$$

Given the two facts  $f_1$  and  $f_2$  we can derive  $+\delta\text{damage}$  and  $+\delta\text{medicalCase}$ . From the unopposed rules for the burdens we obtain  $+\delta B(r_1, \text{tort})$ , (with  $BP$  we mean both  $BPr$  and  $BPe$ ),  $+\delta B(r_1, \text{justification})$ ,  $+\delta B(r_2, \text{crime})$ ,  $+\delta BPr(r_2, \text{justification})$ ,  $BPe(r_2, \neg\text{justification})$ , and  $+\delta B(r_5, \text{damage})$ . For the burden on *negligence* in rule  $r_5$  we have conflicting rules. Rules  $b_{7a0}$  and  $b_{7b0}$  are applicable (i.e., we can prove  $+\delta\text{medicalCase}$ ), and the rules override rules  $b_{5a0}$  and  $b_{7a0}$ , thus for it we conclude  $+\delta B(r_5, \neg\text{negligence})$ . From the burdens just derived, we know that we have to derive  $+\delta\text{tort}$  and  $-\sigma\text{justification}$  to be able to trigger  $r_1$ ; and  $+\delta\text{crime}$  and either  $-\sigma\text{justification}$  or  $+\delta\neg\text{justification}$  to trigger  $r_2$ , and  $+\delta\text{damage}$  and  $-\sigma\neg\text{negligence}$  for  $r_5$ . We do not have information about *threat* thus, we have  $-\sigma\text{threat}$ , which allows us to get  $-\sigma\text{justification}$ . Similarly, we do not have *crime* thus, we have  $-\sigma\text{crime}$  which means  $-\delta\text{crime}$  and thus we have  $-\delta\text{criminalLawLiability}$ . So we cannot establish that

there was criminal law liability. For assessing civil law liability we have to examine the outcome of rule  $r_5$ . We have  $+\partial damage$ , and to prove  $+\partial tort$ , we have to prove  $-\sigma-negligence$ . Since there is no argument (rule) to support that there was no negligence, we derive it. This finally, allow us to derive  $+\partial civilLawLiability$ .

## 6. Conclusion

The purpose of this paper was to illustrate how burdens of proof make sense also within monological argumentation from a knowledge base, and in particular within the framework provided by defeasible logic. We have provided an “objective” characterisation of the burdens of production and persuasion, which refers to the ways a proposition (or its complement) is derivable, abstracting from the agent who is advancing that proposition. On the basis of this characterisation an addition has been provided to rule-based models of norms: it is possible to associate to each element in a rule’s antecedent the required burdens, or to infer burdens from other propositions in the knowledge base. This enables rules to maintain their syntactical structures, even though the proof conditions for the satisfaction of their antecedents are debated or changed. This expands the scope for monological legal argumentation to include the subtleties related to the distinction of the two kinds of burdens. Thanks to the connection with defeasible logic, we have shown that appropriate inferential procedures can be defined, which take into account the different kinds of burdens. This is still very preliminary work. Much remains to be done, in particular in analysing the connection between the model here proposed, semantics for defeasible reasoning and other models for monological and dialogical argumentation.

However, we hope that our attempt can provide an insight into the notion of proof-burdens, and show how this idea can have useful application within systems for defeasible reasoning.

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