Lex minus dixit quam voluit, *lex magis dixit quam voluit*: A formal study on legal compliance and interpretation

Guido Boella[1], Guido Governatori[2], Antonino Rotolo[3], and Leendert van der Torre[4]

[1] University of Turin, Italy
[2] NICTA, Australia
[3] University of Bologna, Italy
[4] University of Luxembourg, Luxembourg

Abstract. This paper argues in favour of the necessity of dynamically restricting and expanding the applicability of norms regulating computer systems like multiagent systems, in situations where the compliance to the norm does not achieve the purpose of the norm. We propose a logical framework which distinguishes between constitutive and regulative norms and captures the norm change power and at the same time the limitations of the judicial system in dynamically revising the set of constitutive rules defining the concepts on which the applicability of norms is based. In particular, the framework is used to reconstruct some interpretive arguments described in legal theory such as those corresponding to the Roman maxims *lex minus dixit quam voluit* and *lex magis dixit quam voluit*. The logical framework is based on an extension of defeasible logic.

1 Introduction and Motivation

An important research issue in AI is to design computer systems whose performance is constrained by suitable sets of legal norms: in this sense, norms establish what legality criteria should apply to their functioning [22]. However, the general idea of regulating computer systems with norms can be modelled in different ways. As, e.g., Boella and van der Torre [4] pointed out in the field of normative multiagent systems, norms may work either as hard or soft constraints. In the first case, computer systems are designed in such a way as to avoid legal violations. In the second case, norms rather provide standards which can be violated, even though any violations should result in sanctions or other normative effects applying to non-compliant agents. To do that, it is necessary to monitor the behaviour of agents and enforce the sanctions.

Soft constraints allow agents to optimize their performance by reasoning about the trade off between respecting the norm – thus incurring in the related compliance costs – and the risk of being sanctioned. However, this additional flexibility of norms as soft constraints is not enough, because it could lead the agent to respect the norm (or otherwise to be sanctioned) even in circumstances where the respect of the norm does not give any advantage to the system, thus wasting his resources while the whole system achieves only a suboptimal state.

Norms are like plans which aim at achieving the social goals the members of a society decided to share [5]. The legislator tries to specify all the circumstances which a norm applies to and all the exceptional contexts where it does not apply, but, as well

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known in the planning community of AI, universal plans rarely are a practicable strategy [7]. An agent should rather produce a partial plan and revise it when part of it becomes unfeasible. In the same way as replanning allows an agent to revise its plans while keeping fixed its original goals, law has a mechanism, called interpretation, to allow norms to be adapted after their creation to the unforeseen situations in order to achieve the social goal they have been planned for. After all, not only the world changes, giving rise to circumstances unexpected to the legislator who introduced the norm, but even the ontology of reality can change with respect to the one constructed by the law to describe the applicability conditions of norms (see, e.g., all the problems concerning the application of existing laws to privacy, intellectual property or technological innovations in healthcare). This adaptation can be made only at the moment of evaluating whether a given behavior in a particular situation should be considered as a violation, i.e., by judges in courts.

Thus, the research question of this paper is: how to formalize the interpretation mechanism of law, so to design more flexible computer systems regulated by norms? This splits in the following subquestions: How does the law model the ontology of concepts to which norms refer to? How can the applicability of norms be restricted or expanded in some situations? How to model the goals associated to norms and how goals are used to evaluate the compliance to a norm in unforeseen circumstances? How to model and at the same time limit the power to interpret norms?

To answer the first subquestion we use the notion of constitutive norms besides regulative ones (henceforth, legal rules). While the latter ones specify the ideal behaviour, the former ones provide an ontology of institutional concepts to which the conditions of legal rules refer to. To model the revision of norms, we use as methodology an extension of Defeasible Logic (DL) [10], which allows us to model constitutive and legal rules and the norm change process, while keeping linear the complexity of the overall process. DL allows us as well to reason about goals assigned to norms and to use them to frame the norm change process within limits.

The layout of the paper is as follows. Section 2 describes the structure of a normative system and the role of interpretation. In Section 3 we explain the distinction between norm restriction and norm expansion together with the role of goals in the interpretation process. Section 4 introduces the variant of DL we use in this paper and Section 5 uses DL to model the norm restriction and norm expansion processes.

2 Legal Rules and Legal Concepts

2.1 The structure of a normative system

As well known, norms have a conditional structure such as $b_1, \ldots, b_n \Rightarrow_O l$ (if $b_1, \ldots, b_n$ hold, then l is obligatory), an agent is compliant with respect to this norm if l holds whenever the obligation l follows from $b_1, \ldots, b_n$. Most logical models of legal reasoning often assume that conditions of norms give a complete description of their applicability (see Sartor [18]). However, this assumption is too strong, due to the complexities and dynamics of the world. Norms cannot take into account all the possible conditions where they should or should not be applied, because the legislator cannot consider all the possible contexts which are exceptional, he cannot foresee unexpected changes of the world and of the ontology of concepts the norms refer to.

Normative systems regulating real societies have two mechanisms to cope with this problem. First they distinguish legal rules (obligations, prohibitions and permissions) from constitutive rules. While the former, which are changed only by the legislative system, specify the ideal behaviour, the latter ones provide, by means of counts-as definitions, an ontology of institutional concepts. The applicability conditions of legal rules very often refer to these institutional concepts, rather than to the so called brute facts.

Second, the judicial system, at the moment in which a case concerning a violation is discussed in court, is empowered to interpret, i.e., to change norms, under some restrictions not to go beyond the purpose from which the legal rules stem. The distinction between legal and constitutive rules (norms vs ontology) suggests that legal interpretation does not amount to revising norms, but to interpreting legal concepts, i.e., to revising constitutive rules [19].

In this paper we adopt the view that the ontology of legal concepts is built via constitutive rules having the so-called counts-as form [20]: $r : a_1, \ldots, a_n \Rightarrow_c b$ For example, a bicycle is considered as a vehicle by the following constitutive rule:

$$r_0 : \mathit{Bike}(x) \Rightarrow_c \mathit{Vehicle}(x)$$

This counts-as rule, if instantiated by any bicycle a, says that a counts as a vehicle.

Constitutive rules have a defeasible character, for example, a bicycle for children cannot be considered as a vehicle:

$$r_1 : \mathit{Bike}(x), \mathit{ForChildren}(x) \leadsto_c \neg \mathit{Vehicle}(x)$$

$$r_0 \succ r_1$$

As usual in DL, our language includes (1) a superiority relation $\succ$ that establishes the relative strength of rules and is used to solve conflicts, (2) special rules marked with $\leadsto$, called defeaters, which are not meant to derive conclusions, but to provide reasons against the opposite.

In general, note that in legal systems counts-as rules may either specify conceptual links between “brute” facts or acts (i.e., non-institutional facts or acts whose status is independent of the existence of any constitutive rule; example: being over 18 years) counts as types of institutional facts or acts (e.g., being adult), or rather specify conceptual links where institutional facts or acts (e.g., a contract made by person j in the name of person k) have the same effects of other institutional facts or acts (e.g., a contract made by k). This view basically implies that the consequents of constitutive rules always correspond to institutional facts or acts. Indeed, constitutive rules are meant to “constitute” and define legal concepts whose existence precisely depends on the existence of constitutive rules. Moreover, there are two sources of constitutive rules, explicit norms like the one defining what means to be adult, but also the usual meaning of the terms, as they appear in a law according to the normal (day to day) meaning and intention of the lawmaker, e.g., ‘Good pater familiae’, ‘Due diligence’.

Here, we will deal with such a type of constitutive rules following the approach described in [11], where it is convincingly argued that an effective way to capture the basic properties of the counts-as link is to reframe it in terms of standard DL.

The set of legal rules is kept to be fixed: any judge during the interpretation process can argue about their applicability conditions but cannot either add new rules nor cancel them. Only legislators have the power to change legal rules.

Legal rules are for example:

$$r_2 : \mathit{Vehicle}(x), \mathit{Park}(y) \Rightarrow_O \neg \mathit{Enter}(x,y)$$

This rule reads as follows: if x is a vehicle and y is a park, then it is (defeasibly) forbidden for any x to enter y.

For the sake of simplicity, we will assume that legal rules only impose duties and prohibitions, and state permissions: they are captured within a suitable extension of standard DL [10].

Finally, as usually assumed in legal theory [16,18], we assign goals to legal rules. In the social delegation cycle [5] norms are planned starting from goals shared by the community of agents. However, such goals play also another role: they pose the limits within which the interpretation process of the judicial systems must stay when interpreting norms.

Note that the goal alone is not sufficient to specify a norm, since there could be many ways to achieve that goal and some guidance should be given to the citizens. Thus, the norm works like a partial plan the legislator set up in advance. The judicial system is left with the task of dynamically adapt the applicability of the regulative norm by revising the constitutive norms referring to its applicability conditions, in order to fulfil the goal of the norm also under unforeseen circumstances. This is why law can be considered as a synecdoche: a term denoting a part of something is used to refer to the whole thing.

In this paper, we define a set Goal of goals and a function $\mathscr{G}$ which maps legal rules into elements of Goal. For example, if $\mathscr{G}(r_2) =$ **road_safety**, this means that the goal of the rule prohibiting to enter into parks is to promote road safety[5]. The idea is quite standard in legal theory [16,18,19] and has been already investigated in AI&Law, even though most works were mainly devoted to case-based reasoning and modeling case-law [2]. Note that, in this paper, goals are considered here as directly specified by the legal rules themselves. In general, the task to determine what goals are supposed to be promoted by rules is usually accomplished by judges by developing suitable arguments during the trial.

As largely acknowledged in legal theory, when it is possible to establish the relative weight of rule goals, this can be used both to determine the relative strength of any legal rule in case of conflicts with other rules and to interpret any legal rule when it is not clear whether this rule can be applied to a given concrete case [16]. As regards the first issue – solving conflicts by referring to rule goals – it seems natural then to define a partial order $>$ over Goal to capture cases where any goal g is more important than any other goal g'. If $g > g'$ then g is more important than g', otherwise they have equal importance. Hence, $>$ may be used to solve conflicts between legal rules. Consider the following rules:

$$r_3 : \mathit{HighWay}(x), \neg \mathit{Authorized\_Area}(x) \Rightarrow_O \neg \mathit{Stop}(x)$$
$$r'_3 : \mathit{HighWay}(x), \neg \mathit{Authorized\_Area}(x), \mathit{Crash}(y) \Rightarrow_O \mathit{Stop}(x)$$

[5] Hereafter, we will use bold type expressions to denote goals.

Rule r_3 states that it is forbidden for drivers to stop in highways except in authorized areas; rule r'_3 says that drivers have to stop when they are responsible for serious car crashes in highways. Suppose that the legal system does not explicitly state what rule should prevail here. If so, resorting to rule goals can help. In fact, we may assume that the goal of r_3 is to promote road safety, while the one of r'_3 is to protect life when it is in serious and imminent danger. Since the latter goal should be more important than the former one, r'_3 will have to prevail over r_3.

This mechanism for solving conflicts will be added in our framework to the standard one adopted in Defeasible Logic [1], which is based on a superiority relation $\succ$ directly applied to rules.

2.2 Revising constitutive rules

Checking legal compliance requires to establish if a legal rule $r : b_1, \ldots, b_n \Rightarrow_O \neg l$ is violated by a fact or action l happened under some circumstances H. Let us assume that r states that $\neg l$ ought to be the case. However, l is not necessarily a violation, because we also have to check whether H, via the constitutive rules, matches with the applicability conditions $b_1, \ldots, b_n$ of r. In easy cases, these match and l directly amount to a violation. However, jurists argue that we have cases where this does not hold, as for example when there is a discrepancy between the literal meaning of $b_1, \ldots, b_n$ and the goal assigned to the rule r by the legislator. If so, even though H matches with $b_1, \ldots, b_n$, we do not have a violation because H should not match with $b_1, \ldots, b_n$. A non-literal interpretation of $b_1, \ldots, b_n$ would exclude H as a circumstance falling within the scope of r, since the goal of the norm would be achieved anyway: *lex magis dixit quam voluit*, the law said more than what the legislator was meaning to say. Analogously, not all cases in which H mismatches with $b_1, \ldots, b_n$ are not violations. We could have that *lex minus dixit quam voluit*, the law said less than what the legislator was meaning to say: here a non-literal, goal-based interpretation of r would lead to broaden its applicability scope to match H, thus making the agent a violator [16].

3 Interpreting Legal Rules

In this section we describe the interpretation process using a running example, first considering a scenario of norm restriction and second a norm expansion.

Suppose Mary enters a park with her bike, thus apparently violating rule r_2 above about vehicles' circulation. Police stops her when she is still on her bike in the park and fines her. Mary thinks this is unreasonable and sues the municipality because she thinks that here the category "vehicle" should not cover bikes.

3.1 Restricting the applicability of norms

In the first case the conceptual domain of the normative system, corresponding to a set of constitutive rules, allows us to derive that any bike a is indeed a vehicle. The goal of the norm r_2 is reducing pollution $\mathscr{G}(r_2) = \neg\textbf{pollution}$. In court, the judge has to establish if Mary violated r_2 or not.

If T is the case, the judge could argue that Mary should be fined, as r_2 clearly applies to her:

$$T = \{r_0 : \mathit{Bike}(x) \Rightarrow_c \mathit{Vehicle}(x),$$
$$r_4 : \mathit{2\_wheels}(x), \mathit{Transport}(x), \neg \mathit{Engine} \Rightarrow_c \mathit{Bike}(x)\}$$

But suppose that the judge can show that, if Mary's case fulfils the applicability conditions of r_2 (Mary's bike is a vehicle) then a goal which is incompatible with the goal assigned to r_2 would be promoted. Since $\mathscr{G}(r_2) = \neg\textbf{pollution}$, prohibiting to circulate with bikes in parks would encourage people to get around parks by car and then walk. This would be against the goal of r_2 and so the judge has good reasons to exclude that bikes are vehicles when r_2 should be applied. Accordingly, when arguing in this way, the judge may interpret r_2 by reducing its applicability conditions as far as Mary's case is concerned, and so by contracting T in order to obtain in T that Mary's bike is not a vehicle in the context of the current situation.

3.2 Expanding the applicability of norms

Alternatively, the conceptual domain could exclude that bikes are vehicles and the goal of r_2 could be the safety of people walking in the park **pedestrian_safety**:

$$T' = \{r_4 : \mathit{2\_wheels}(x), \mathit{Transport}(x), \neg \mathit{Engine}(x) \Rightarrow_c \mathit{Bike}(x),$$
$$r_5 : \mathit{Bike}(x) \Rightarrow_c \neg \mathit{Vehicle}(x),$$
$$r_6 : \mathit{Transport}(x) \rightsquigarrow_c \mathit{Vehicle}(x)\}$$
$$\succ = \{r_5 \succ r_6\}$$

T' also includes r_6, which states that, if we know that some x has purpose of transport, then we have reasons to block other rules which would lead to exclude that x is a vehicle. However, in T' r_6 is made weaker than r_5 via the superiority relation $\succ$, and so, if x is a bike, we conclude by r_5 that x is not a vehicle.

Now, suppose the judge has to settle Mary's case starting from T'. Again, the goal of legal rules such as r_2 may be decisive. The judge could argue that Mary should not be fined, as r_2 clearly does not apply. But suppose that, since r_2 is not fulfilled, this would be against the goal of r_2, which is now **pedestrian_safety**. In this case, the judge has rather good reasons to consider bikes as vehicles when r_2 is concerned. Hence, the judge may interpret r_2 by broadening its applicability conditions as far as Mary's case is concerned, and so by revising T' in such a way as Mary's bike is a vehicle.

3.3 Constraints on the revision

In general, we should note that such types of revisions have to satisfy some requirements (let's still bear in mind the case of Mary's bike):

1. there is no other $g' \in$ Goal such that
   - the revision of T (or of T') promotes r_2's goal g which is incompatible, in the application context of r_2, with respect to the goal g' of another applicable rule r, and

– $\mathscr{G}(r_2) \not> \mathscr{G}(r)$ ($\mathscr{G}(r_2)$ is not more important than $\mathscr{G}(r)$);

2. our set of constitutive rules should suggest us that the concept of *Bike* can be subsumed under the concept of *Vehicle*;

Point 1 above states that, if by contracting or revising the concept of *Bike*, we undermine at least one equally or more important goal, which is supposed to be promoted by another applicable rule, then such a contraction or revision is not acceptable. This limit is well-known by lawyers and legal theorists [18,16], who often argue that any legal interpretation should be coherent within the legal system as a whole.

Point 2 above is rather connected with the fact that the set of constitutive rules should inherently provide some conceptual limits for any interpretation. Indeed, suppose that Mary enters the park with a gun. We could have reasons for arguing that entering with a gun is dangerous for all people in the park, and so for pedestrians too. However, this is not enough, of course, for arguing that guns are vehicle. In other words, if we do not have any other legal rules prohibiting to enter parks with guns, this behaviour will be permitted. Hence, point 2 has to do with Hart's [15] theory of penumbra: we have a core of cases which can be clearly classified as belonging to the legal concept and a penumbra of hard cases, whole membership in the concept can be disputed; but hard cases should exhibit some conceptual link with the core of cases. This idea is formally captured here by confining the revision of the set of constitutive rules only to those situations where such a set, though failing to prove that a bike is a vehicle, already contains reasoning chains suggesting that this may be the case. For example, if we have

$$r_4 : \mathit{2\_wheels}(x), \mathit{Transport}(x), \neg \mathit{Engine}(x) \Rightarrow_c \mathit{Bike}(x)$$
$$r_7 : \mathit{Bike}(x) \rightsquigarrow_c \mathit{Vehicle}(x)$$

r_7 states that, if we know that some x is a bike, this is not sufficient to prove that x is a vehicle (r_7 is a defeater), but it is sufficient to block other rules which would lead to exclude that x is a vehicle. This means that, possibly, if x is a bike, then it could not be unreasonable to consider x as a vehicle (for a similar reading of defeaters in terms of $\Diamond$, but applied to the concept of permission, see [10,12]). Hence, the revision would require, for example, that r_7 is replaced by

$$r_0 : \mathit{Bike}(x) \Rightarrow_c \mathit{Vehicle}(x)$$

The framework we have informally depicted above suggests that we also need a logical component to reason about goals. Such a component should enable us to check whether some situations promote goals or their negations. For our purpose it is sufficient to introduce a suitable set of rules for goals [11] which should be used to establish what are the effects of situations where legal rules are violated or complied with, and, in doing so, to see whether they are consistent with the goals. In other words, we have to devise a set of rules like $d_1, \ldots, d_n \Rightarrow_G e$: if applicable in a given context, this rule allows for deriving $G\,e$, meaning that e is a goal promoted by the underlying normative theory D. Consider once again rule r_2; suppose that its goal is **pedestrian_safety** and that Mary's case is described by the following set H of facts:

$$H = \{\mathit{Bike}(b),\ \mathit{Park}(p),\ \mathit{Enter}(b,p),\ \mathit{NarrowSpace}(p),\ \mathit{UnprotectedChildArea}(p)\}$$

H states that Mary enters the park p with her bike b. The park has narrow spaces for walking and there are unprotected children's play areas. This set assumes that r_2 is violated, at least in the hypothetical perspective in which Mary should not enter.

Suppose now that rules for goals correspond to the following set:

$$R^G = \{r_8 : Bike(x), Park(y), Enter(x,y) \Rightarrow_G \textbf{fast\_circulation},$$
$$r_9 : NarrowSpace(x), UnprotectedChildArea(x),$$
$$G\,\textbf{fast\_circulation} \Rightarrow_G \neg\textbf{pedestrian\_safety}\}$$

Rule r_8 states that entering parks with bikes promotes the fast circulation of people in those parks; rule r_9 says that, if fast circulation is promoted (i.e., it is possible to derive as goal of the normative theory **fast_circulation**, that is, G**fast_circulation**) and parks have narrow spaces and unprotected children's play areas, then the promoted goal is the negation of pedestrians safety. If so, if Mary's bike is allowed to enter, then we would promote a goal which is incompatible with the goal of r_2.

4 The Logical Framework

The following framework is an extension of DL; such an extension is in line with works such as [10,11]. In particular, on account of the informal presentation given in the previous section, while counts-as rules do not prove modalised literals, the system develops a constructive account of those modalities that rather correspond to obligations and goals: rules for these concepts are thus meant to devise suitable logical conditions for introducing modalities. For example, while a counts-as rule such as $a_1, \dots, a_n \Rightarrow_c b$, if applicable, will basically support the conclusion of b, rules such as $a_1, \dots, a_n \Rightarrow_O b$ and $d_1, \dots, d_n \Rightarrow_G e$ if applicable, will allow for deriving $O\,b$ and $G\,e$, meaning the former that b is obligatory, the latter that e is a goal promoted by the facts used to derive it (as previously explained).

Note that the framework is restricted to essentially propositional DL. Indeed, rules with free variables are interpreted as rule schemas, that is, as the set of all ground instances; in such cases we assume that the Herbrand universe is finite. This assumption is harmless in this context, as the rule applicability domains at hand always refer to finite set of individuals.

In our language, for $X \in \{c, O, G\}$, *strict rules* have the form $\phi_1, \dots, \phi_n \rightarrow_X \psi$. *Defeasible rules* have the form $\phi_1, \dots, \phi_n \Rightarrow_X \psi$. A rule of the form $\phi_1, \dots, \phi_n \leadsto_X \psi$ is a *defeater*. A rule can be understood as a binary relationship between a set of premises and a conclusion. Accordingly, the mode determines the type of conclusion one can obtain, and the three types of rules establish the strength of the relationship. Strict rules provide the most stronger connection between a set of premises and their conclusion: whenever the premises are deemed as indisputable so is the conclusion; then we have defeasible rules: a defeasible rules, given the premises, allows us to derive the conclusion unless there is evidence for its contrary; finally we have defeaters. A defeater suggests that there is a connection between its premises and the conclusion, but this connection is not strong enough to warrant the conclusion on its own; on the other hand a defeater shows that there is some evidence for the conclusion, thus it can be used to defeat rules for the opposite conclusion.

Definition 1 (Language). *Let* PROP *be a set of propositional atoms,* Goal *be a set of goal atoms,* $\mathrm{MOD} = \{c, O, G\}$*, and* Lbl *be a set of labels. The sets defined below are the smallest sets closed under the given construction conditions:*

Literals and goals

$$\mathrm{Lit} = \mathrm{PROP} \cup \{\neg p | p \in \mathrm{PROP}\}$$
$$\mathrm{GoalLit} = \mathrm{Goal} \cup \{\neg g | g \in \mathrm{Goal}\}$$

If q is a literal or a goal, $\sim q$ *denotes the complementary literal or goal (if q is a positive literal or goal p then* $\sim q$ *is* $\neg p$*; and if q is* $\neg p$*, then* $\sim q$ *is p);*

Modal literals and modal goals

$$\mathrm{ModLit} = \{Ol, \neg Ol | l \in \mathrm{Lit}\}$$
$$\mathrm{ModGoal} = \{Gl, \neg Gl | g \in \mathrm{GoalLit}\};$$

Rules $\mathrm{Rul} = \mathrm{Rul}_{\mathrm{s}} \cup \mathrm{Rul}_{\mathrm{d}} \cup \mathrm{Rul}_{\mathrm{dft}}$*, where* $X \in \{c, O\}$ *and* $\mathrm{Rul}_{\mathrm{s}} = \mathrm{Rul}^X_{\mathrm{s}} \cup \mathrm{Rul}^G_{\mathrm{s}}$*,* $\mathrm{Rul}_{\mathrm{d}} = \mathrm{Rul}^X_{\mathrm{d}} \cup \mathrm{Rul}^G_{\mathrm{d}}$*, and* $\mathrm{Rul}_{\mathrm{dft}} = \mathrm{Rul}^X_{\mathrm{dft}} \cup \mathrm{Rul}^G_{\mathrm{dft}}$ *such that*

$$\mathrm{Rul}^X_{\mathrm{s}} = \{r : \phi_1, \ldots, \phi_n \rightarrow_X \psi | r \in \mathrm{Lbl}, A(r) \subseteq \mathrm{Lit}, \psi \in \mathrm{Lit}\}$$
$$\mathrm{Rul}^G_{\mathrm{s}} = \{r : \phi_1, \ldots, \phi_n \rightarrow_G \psi | \; r \in \mathrm{Lbl}, A(r) \subseteq \mathrm{Lit} \cup \mathrm{ModLit} \cup \mathrm{ModGoal}, \psi \in \mathrm{GoalLit}\}$$
$$\mathrm{Rul}^X_{\mathrm{d}} = \{r : \phi_1, \ldots, \phi_n \Rightarrow_X \psi | r \in \mathrm{Lbl}, A(r) \subseteq \mathrm{Lit}, \psi \in \mathrm{Lit}\}$$
$$\mathrm{Rul}^G_{\mathrm{d}} = \{r : \phi_1, \ldots, \phi_n \Rightarrow_G \psi | \; r \in \mathrm{Lbl}, A(r) \subseteq \mathrm{Lit} \cup \mathrm{ModLit} \cup \mathrm{ModGoal}, \psi \in \mathrm{GoalLit}\}$$
$$\mathrm{Rul}^X_{\mathrm{dft}} = \{r : \phi \rightsquigarrow_X \psi | r \in \mathrm{Lbl}, A(r) \subseteq \mathrm{Lit}, \psi \in \mathrm{Lit}\}$$
$$\mathrm{Rul}^G_{\mathrm{dft}} = \{r : \phi_1, \ldots, \phi_n \rightsquigarrow_G \psi | \; r \in \mathrm{Lbl}, A(r) \subseteq \mathrm{Lit} \cup \mathrm{ModLit} \cup \mathrm{ModGoal}, \psi \in \mathrm{GoalLit}\}$$

We use some obvious abbreviations, such as superscripts for the rule mode (c, G, O)*, subscripts for the type of rule, and* $\mathrm{Rul}[\phi]$ *for rules whose consequent is* ϕ*, for example:*

$$\mathrm{Rul}^c = \{r : \phi_1, \ldots, \phi_n \hookrightarrow_c \psi | \hookrightarrow \in \{\rightarrow, \Rightarrow, \rightsquigarrow\}\}$$
$$\mathrm{Rul}_{\mathrm{sd}} = \{r : \phi_1, \ldots, \phi_n \hookrightarrow_X \psi | X \in \mathrm{MOD}, \hookrightarrow \in \{\rightarrow, \Rightarrow\}\}$$
$$\mathrm{Rul}_{\mathrm{s}}[\psi] = \{\phi_1, \ldots, \phi_n \rightarrow_X \psi | X \in \mathrm{MOD}\}$$

We use $A(r)$ *to denote the set* $\{\phi_1, \ldots, \phi_n\}$ *of* antecedents *of the rule r, and* $C(r)$ *to denote the* consequent ψ *of the rule r.*

A normative theory is the knowledge base which is used to reason about the applicability of legal rules included in the theory itself.

Definition 2 (Normative Theory). *A normative theory is a structure*

$$D = (F, G, R^c, R^O, R^G, \succ, \mathscr{G}, >)$$

where

– $F \subseteq \mathrm{Lit} \cup \mathrm{ModLit} \cup \mathrm{ModGoal}$ *is a finite set of facts;*

– $G \subseteq \text{GoalLit}$ *is a set of rule goals,*
– $R^c \subseteq \text{Rul}^c$ *is a finite set of counts-as rules,*
– $R^O \subseteq \text{Rul}^O$ *is a finite set of obligation rules,*
– $R^G \subseteq \text{Rul}^G$ *is a finite set of goal rules,*
– $\succ$ *is an acyclic relation (called superiority relation) defined over* $(R^c \times R^c) \cup (R^O \times R^O) \cup (R^G \times R^G)$*,*
– $\mathscr{G} : R^O \mapsto G$ *is a function assigning a goal to each obligation rule,*
– $>$ *is a partial order over G defining the relative importance of the rule goals.*

Proofs are sequences of literals and modal literals together with the so-called proof tags $+\Delta$, $-\Delta$, $+\partial$ and $-\partial$. If $X \in \{c, O, G\}$, given a normative theory D, $+\Delta^X q$ means that literal q is provable in D using only facts and strict rules for X, $-\Delta^X q$ means that it has been proved in D that q is not definitely provable in D, $+\partial^X q$ means that q is defeasibly provable in D, and $-\partial^X q$ means that it has been proved in D that q is not defeasibly provable in D.

Definition 3. *Given a normative theory D, a proof in D is a linear derivation, i.e, a sequence of labelled formulas of the type* $+\Delta^X q$*,* $-\Delta^X q$*,* $+\partial^X q$ *and* $-\partial^X q$*, where the proof conditions defined in the rest of this section hold.*

Definition 4. *Let D be a normative theory. Let* $\# \in \{\Delta, \partial\}$ *and* $X \in \{O, G\}$*, and* $P = (P(1), \dots, P(n))$ *be a proof in D. A literal q is* #-provable *in P if there is a line* $P(m)$*,* $1 \leq m \leq n$*, of P such that either*

1. *q is a literal and* $P(m) = +\#^c q$ *or*
2. *q is a modal literal or a modal goal* Xp *and* $P(m) = +\#^X p$ *or*
3. *q is a modal literal or a modal goal* $\neg Xp$ *and* $P(m) = -\#^X p$*.*

A literal q is #-rejected *in P if there is a line* $P(m)$ *of P such that*

1. *q is a literal and* $P(m) = -\#^c q$ *or*
2. *q is a modal literal or a modal goal* Xp *and* $P(m) = -\#^X p$ *or*
3. *q is a modal literal or a modal goal* $\neg Xp$ *and* $P(m) = +\#^X p$*.*

The definition of Δ^X, $X \in \{c, O, G\}$ describes just forward (monotonic) chaining of strict rules[6]:

$+\Delta^X$: If $P(n+1) = +\Delta^X q$ then
(1) $q \in F$ if $X = c$ or $Xq \in F$ or
(2) $\exists r \in R^X_s[q] :\ \forall a \in A(r)$ a is Δ-provable.

To show that a literal q is defeasibly provable with the mode X we have two choices: (1) We show that q is already definitely provable; or (2) we need to argue using the defeasible part of a normative theory D. For this second case, some (sub)conditions must be satisfied. First, we need to consider possible reasoning chains in support of

[6] For space reasons, in the remainder we present only the proof conditions for $+\Delta$ and $+\partial$. Conditions for the negative tags are obtained using the so-called principle of strong negation and the notion of #-*rejected* [10].

$\sim q$ with the mode X, and show that $\sim q$ is not definitely provable with that mode (2.1 below). Second, we require that there must be a strict or defeasible rule with mode X for q which can be applied (2.2 below). Third, we must consider the set of all rules which are not known to be inapplicable and which permit to get $\sim q$ with the mode X (2.3 below). Essentially, each such a rule s attacks the conclusion q. For q to be provable, s must be counterattacked by a rule t for q with the following properties: (i) t must be applicable, and (ii) t must prevail over s. Thus each attack on the conclusion q must be counterattacked by a stronger rule. In other words, r and the rules t form a team (for q) that defeats the rules s. Note that in our framework, in addition to $\succ$, also goals can be used to determine the relative strength of any legal rule in case of conflicts with other rules; the following definition enables us to handle together goal preferences and the superiority relation $\succ$.

Definition 5. *Let $D = (F, G, R^c, R^O, R^G, \succ, \mathscr{G}, >)$ be a normative theory. A rule r* prevails *over another rule s iff*

- *$\mathscr{G}(r) > \mathscr{G}(s)$ or*
- *$r \prec s$ and $\mathscr{G}(s) \not> \mathscr{G}(r)$*

$+\partial^X$: If $P(n+1) = +\partial^X q$ then
(1)$+\Delta^X q \in P(1..n)$ or
(2) (2.1) $-\Delta^X \sim q \in P(1..n)$ and
(2.2) $\exists r \in R^X_{\mathrm{sd}}[q]$ such that $\forall a \in A(r)$ a is ∂-provable, and
(2.3) $\forall s \in R^X[\sim q]$ either $\exists a \in A(s)$ such that a is ∂-rejected, or
(2.3.1) $\exists t \in R^X[q]$ such that $\forall a \in A(r)$ a is ∂-provable and
t prevails over s

Definition 6. *Given a normative theory D, $D \vdash \pm\#^X l$ (i.e., $\pm\#^X l$ is a conclusion of D), where $\# \in \{\Delta, \partial\}$ and $X \in \{c, O, G\}$, iff there is a proof $P = (P(1), \ldots, P(n))$ in D such that $P(n) = \pm\#^X l$.*

It is worth noting that our logic enjoys nice computational properties:

Theorem 1. *For every normative theory D, the conclusions of D can be computed in time linear to the size of the theory, i.e., $O(|U^D| * |R|)$, where U^D is the Herbrand base of the normative theory D.*

Proof. The proof comes directly from the result provided in [11,10]. In fact, the current logic is structurally similar to those presented there.

5 A Framework for Revising Constitutive Rules

5.1 Legal Compliance: Why Interpret?

The informal discussion presented in Section 3 requires to formally characterise those situations where a context and an action or fact make a norm applicable, fulfil it and those situations where the behaviour of the agent either promotes or undermines the goal of a norm.

Definition 7 (Context). *A context H is a set of literals $\{f_1, \dots, f_m\}$.*

The context just identifies a set of literals 'relevant' for a particular case. In particular we will use the context as the set of factors used to determine whether we need to extend or restrict the interpretation to terms. Another possible use of the context is to set up hypothetical scenarios.

Definition 8 (Rule Applicability). *Given a context H, let*

$$D = (F \cup H, G, R^c, R^O, R^G, \succ, \mathscr{G}, >)$$

be a normative theory.

A rule $r \in R^O$ is applicable *in D iff, for all $b \in A(r)$, b is ∂^c-provable in D.*

Remark 1. The notion of rule applicability is technically straightforward. It expresses the idea that a norm r is applicable iff all the antecedents of r are provable using the constitutive rules of the theory. Notice that we focus only to the case where the antecedents are defeasibly provable (∂-provable). This does not mean that, given the rule r, if the antecedents are indisputably provable (Δ-provable), the rule is not applicable at all. Quite the contrary. The reason why we work only on defeasible provability is that, in our framework, the process of revision of the applicability conditions of a rule is possible only when the antecedents are defeasibly derivable. Indeed, as we will see, blocking the applicability of a rule requires resorting to defeaters and so can be done only when the applicability of a rule is not indisputable (defeaters are weaker in our logic than strict rules).

We now introduce the notion of compliance to a norm and violation:

Definition 9 (Rule Fulfilment and Violation). *A normative theory*

$$D = (F, G, R^c, R^O, R^G, \succ, \mathscr{G}, >)$$

and a context H fulfil *$r \in R^O_{\mathrm{sd}}$ iff,*

- *if we have that*

$$D' = (F \cup H, G, R^c, R^O, R^G, \succ, \mathscr{G}, >) \vdash +\partial^O C(r)$$

- *r is applicable in D'*

then, either

- *$D' \vdash +\partial^c l$ when $C(r)$ is a positive literal l (r is a conditional obligation) or*
- *$D' \vdash -\partial^c l$ or $D' \vdash +\partial^c \neg l$ when $C(r)$ is a negative literal $\neg l$ (r is a conditional prohibition).*

D and H violate *the rule r whenever D and H do not fulfil r.*

Remark 2. For the notion of rule fulfillment and violation it is not sufficient for a norm to be applicable, but the conclusion of the norm should be derivable $+\partial^O C(r)$ too, since other rules with higher priority could defeat it. In this case, the norm is fulfilled if the object of the norm l is defeasibly derivable, $+\partial^c l$, (or the contrary is not derivable, $-\partial^c l$ in case of prohibitions, i.e., obligations concerning a negated literal $\neg l$). It is violated in the opposite case.

Example 1. Suppose we have a context H with the following facts:

$$H = \{Park, Bike, Enter\}$$

The set of constitutive rules contains only the following

$$r_0 : Bike \Rightarrow_c Vehicle$$

while the set of legal rules only includes

$$r_2 : Vehicle, Park \Rightarrow_O \neg Enter$$

Clearly, rule r_2 is violated because it is applicable, we can obtain $+\partial^O \neg Enter$ (rule r is not defeated) but we have *Enter*.

Before revising a rule, the judge has to consider whether the goal of the norm has been violated or promoted.

Definition 10 (Goal Violation and Promotion). *A normative theory*

$$D = (F, G, R^c, R^O, R^G, \succ, \mathscr{G}, >)$$

and the context $H = \{f_1, \ldots, f_m\}$ violate the goal g *of* $r : b_1, \ldots, b_n \hookrightarrow_O l \in R^O_{\text{sd}}$ *iff*

$$(F, G, R^c, R^O, R^G, \succ, \mathscr{G}, >) \vdash -\partial^G \neg g$$
$$(F \cup H, G, R^c, R^O, R^G, \succ, \mathscr{G}, >) \vdash +\partial^G \neg g$$

D and H promote the goal g *of* r *iff*

$$(F, G, R^c, R^O, R^G, \succ, \mathscr{G}, >) \vdash -\partial^G g$$
$$(F \cup H, G, R^c, R^O, R^G, \succ, \mathscr{G}, >) \vdash +\partial^G g$$

Remark 3. The goal is violated if the opposite goal is derived, while it is promoted if it follows from the normative theory using the goal rules. Note that to verify whether the goal of the norm is violated, it is not sufficient that the opposite goal is defeasibly derived from the context ($+\partial^G \neg g$), but it is necessary to verify that from the normative theory itself does not defeasibly derive the opposite goal. The same must be done for goal promotion.

Example 2. Let us assume to have the rules of Example 1 and the following context H:

$$H = \{Park, Bike, Enter\}$$

	Applicable	Rule Fulfilment	Rule Goal	What-if	Judiciary	Law-making
0	no	no	no	no		ineffective
1	no	no	no	yes	**expand**, violation	restrictive
2	no	no	yes	no		contradictory
3	no	no	yes	yes		superfluous
4	no	yes	no	no		like 0
5	no	yes	no	yes		like 2
6	no	yes	yes	no		**expand**
7	no	yes	yes	yes		like 3
8	yes	no	no	no	**restrict**	ineffective
9	yes	no	no	yes	violation	
10	yes	no	yes	no	**restrict**	like 2
11	yes	no	yes	yes	**restrict**	superfluous
12	yes	yes	no	no		ineffective
13	yes	yes	no	yes	contradictory	**restrict**
14	yes	yes	yes	no		
15	yes	yes	yes	yes		superfluous

Table 1. Violation and Interpretation

Suppose now that the goal of r_2 is **pedestrian_safety**. The set of goal rules consists of

$$r_{10} : Bike, Park, \neg Enter \Rightarrow_G \textbf{pedestrian\_safety}$$
$$r'_{10} : Bike, Park, Enter \Rightarrow_G \neg\textbf{pedestrian\_safety}$$

We can apply rule r'_{10} and so we obtain $+\partial^G \neg$**pedestrian_safety**, which could not be proved if we did not consider the context H. Hence, H and D violate the goal of r_2. On the contrary, if H contains $\neg Enter$ instead of $Enter$, we obtain $+\partial^G$**pedestrian_safety**, thus promoting the goal of r_2.

The concepts above are sufficient to define an exhaustive taxonomy of cases among which we can identify those that require the restriction or expansion of the applicability conditions of legal rules.

Let us consider in Table 1 a normative theory D and a legal rule $r : b_1, \ldots, b_n \hookrightarrow_O l$ ($\hookrightarrow \in \{\rightarrow, \Rightarrow\}$) in it, such that the goal of r is g. As informally discussed before, we have to assess if $\sim l$ and a certain set of circumstances H amount to a violation of r or not. There are several parameters to consider: the applicability of the norm, its fulfilment or violation (whether we have l or $\sim l$), and the satisfaction or not of the goal associated to the norm. Besides these factors, we should consider for each combination whether a different behaviour of the agent would result in a situation which is advantageous for the law or not (column "What if" considers the goal satisfaction assuming the opposite of what indicated in the column Rule Fulfilment)[7]. The last two columns classify the cases under two dimensions: first ("judiciary"), the judge should or should not change the applicability conditions of the norm in the case under exam

[7] In other words, if the column "Rule Fulfilment" indicates "no" ($\sim l$), the column "What-if" indicates whether the rule goal is promoted by having l, and viceversa

of the judicial system (with the two subcases of restricting or expanding applicability conditions); second ("Law-making"), the norm is not adequate from the juridical point of view (the legislator should change it).

We focus on the cases which are relevant for the judiciary point of view: the agent fined by the police goes to a court to defend his case. Case 1 is the prototypical case for expansion: the norm is not applicable but if it were applicable, the agent would have violated it by not achieving l. This behaviour does not satisfy the goal g of the norm, so considering the agent a violator would be useless. Before deciding that it is necessary to enlarge the norm applicability conditions to cover the current case, the judge must make a what-if hypothetical analysis and verify that the goal would have been achieved if the agent complied with the norm. Otherwise we have just an ineffective norm, whose satisfaction does not contribute to the goal (see Case 0). Case 6 requires expansion too, but only from the point of view of the legislator, since the agent is fulfilling the norm.

The situation for norm restriction is more complicated. In Case 8 the agent should considered as a violator, since he did not respect an applicable norm. His behaviour does not achieve the goal of the norm, but before declaring him a violator the judge has to check if a compliant behaviour would have achieved the goal of the norm. If yes, the agent is really violating the norm (Case 9), otherwise the norm must be restricted since in the current situation the norm is useless (Case 8). Cases 10 and 11 do not instead require a what-if analysis since the agent is achieving the goal even if he violates the norm. In both cases the judge declares the agent non violator and restricts the applicability conditions of the norm to exclude the current situation. The two cases differ only from the juridical point of view, but discussing this dimension is beyond the scope of the paper.

5.2 Revising the Rule Scope

Before formally presenting the operations of expansion and contraction of the applicability conditions of rules, let us introduce an auxiliary concept, which is needed to identify possible reasoning chains leading to a conclusion we want to contract (in the case *Lex magis dixit quam voluit*), or reinstate or introduce (in the case *Lex minus dixit quam voluit*).

Definition 11 (Reasoning chains). *Let*

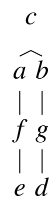
$$D = (F, G, R^c, R^O, R^G, \succ, \mathscr{G}, >)$$

be a normative theory. A counts-as reasoning chain $\mathscr{C}$ in D for a literal l is a finite sequence $\mathscr{R}_1, \ldots, \mathscr{R}_n$ where

- $\mathscr{R}_i \subseteq R^c$, $1 \leq i \leq n$,
- $\mathscr{R}_n = \{a_1, \ldots, a_m \hookrightarrow_c l | \hookrightarrow \in \{\rightarrow, \Rightarrow, \leadsto\}\}$,
- $\mathscr{R}_k \subseteq R^c$, $1 < k \leq n$, *is such that* $\forall\, r^k \in \mathscr{R}_k$, $\forall b \in A(r^k) : \exists r^{k-1} \in \mathscr{R}_{k-1} : b = C(r^{k-1})$.

For all $s \in \mathscr{R}_i$, $1 \leq i \leq n$, we will say that s is in $\mathscr{C}$. If a literal p occurs in the head or the body of any s in $\mathscr{C}$, we will say that p is in $\mathscr{C}$. We define analogously a goal or an obligation reasoning chain $\mathscr{C}$ in D for a literal l when all rules in $\mathscr{C}$ are in R^G or R^O, respectively.

Remark 4. The concept of reasoning chain is nothing but the proof-theoretic version of the notion of argument for a literal l in the Argumentation Semantics for DL [8]. Consider the following example:



$$
\begin{aligned}
&r : e \Rightarrow_c f\\
&s : f \Rightarrow_c a\\
&t : d \Rightarrow_c g\\
&u : g \Rightarrow_c b\\
&w : a, b \Rightarrow_c c
\end{aligned}
$$

The tree on the left side is the argument for the literal c we can build using the rules r, s, t, u, w on the right side (provided that e and d hold). The structure can be reframed in terms of a reasoning chain: $\mathscr{R}_n$, $n = 3$, contains the rule w; $\mathscr{R}_{n-1}$ contains the rules s and u; finally, $\mathscr{R}_{n-2}$ contains r and t. Note that the main difference between the notion of argument and that of reasoning chain is that we allow in reasoning chains to have defeaters in any sets, whereas the arc of an argument can correspond to a defeater only if it is at the top of the tree. Indeed, a defeater cannot prove anything but only block conclusions. On the contrary, we want to select here also those chains in which defeaters can be replaced, for instance, by defeasible rules in order to reinstate or introduce new conclusions (*Lex minus dixit quam voluit*).

We are now ready to formally define the operations of expansion (*Lex minus dixit quam voluit*) and contraction (*Lex magis dixit quam voluit*) of the applicability conditions of a norm corresponding to the cases 1, 8, 10, and 11 of Table 1. The following operations thus implement the intuitions we have illustrated above and in Section 3.

Definition 12 (Rule Expansion). *Let*

$$D = (F, G, R^c, R^O, R^G, \succ, \mathscr{G}, >)$$

be a normative theory, $r : b_1, \ldots, b_n \hookrightarrow_O l \in R^O$ be a legal rule, and H be a context. If

1. *$\{b_k, \ldots b_{k+j}\} \subseteq A(r)$ and, for all b_t, $k \leq t \leq k+j$,*
$$(F \cup H, G, R^c, R^O, R^G, \succ, \mathscr{G}, >) \not\vdash +\partial^c b_t$$
2. *D and $H \cup \{\sim l\}$ violate the goal g of r, and*
3. *D and $H \cup \{l\}$ promote the goal g of r, and*
4. *there exist the counts-as reasoning chains $\mathscr{C}_k, \ldots \mathscr{C}_{k+j}$ in D for $b_k, \ldots b_{k+j}$, such that for each $f \in H$, f is in $\mathscr{C}_h$, $k \leq h \leq k+j$,*

then the expansion of the applicability conditions of a legal rule r with respect to the context H corresponds to the following operation $D^*_{b_k,\ldots,b_{k+j}}$ over D:

$$D^*_{b_k,\ldots,b_{k+j}} = (F, G, R'^c, R^O, R^G, \succ', \mathscr{G}, >)$$

where

$$
\begin{aligned}
R'^c &= R^c - \{r' : d_1, \ldots, d_n \leadsto_c e \mid r' \text{ is in } \mathscr{C}_h\} \cup \{r' : d_1, \ldots, d_n \Rightarrow_c e\}\\
\succ' &= (\succ \cup \{r' \succ s \mid r' \text{ is in } \mathscr{C}_h, s \in R^c[\sim C(r')]\}) - \{t \succ r' \mid t \in R^c[\sim C(r')]\}
\end{aligned}
$$

such that

$$D' = (F \cup H, G, R'^c, R^O, R^G, \succ', \mathscr{G}, >) \vdash -\partial_G \neg g'$$

where g' is the goal of any rule $z \in R^O_{\text{sd}}$ applicable in D' such that $g \not> g'$.

Remark 5. This definition considers the situation where a norm r is only partially applicable since some of the conditions cannot be derived from the facts F and context H (1). However, in this situation, not respecting the norm (i.e., $\sim l$) leads to a violation of the goal g of norm r (2). This violation of the goal would be avoided by complying with the norm, i.e., by achieving l (3). Moreover, the constitutive rules suggest that some elements of the context could be interpreted as the missing applicability conditions of the rule r (4). Thus, the normative theory should be expanded by transforming some defeater rules among the constitutive rules, in defeasible rules. The resulting normative theory is however subject to the final constraint that no other more important goal g' of any norm would be violated ($-\partial_G \neg g'$).

Example 3. Consider the following normative theory augmented with the context H regarding Mary's case[8]:

$$F = \{\mathit{Park},\ \mathit{UnprotectedChildArea},\ \mathit{NarrowSpace}\}$$
$$H = \{\mathit{2\_wheels},\ \mathit{Transport},\ \neg \mathit{Engine}\}$$
$$G = \{\textbf{fast\_circulation},\ \textbf{pedestrian\_safety}\}$$
$$R^c = \{r_4 : \mathit{2\_wheels},\ \mathit{Transport},\ \neg \mathit{Engine} \Rightarrow_c \mathit{Bike},$$
$$r_7 : \mathit{Bike} \leadsto_c \mathit{Vehicle},$$
$$r_{11} : \mathit{Transport}, \neg \mathit{Engine} \Rightarrow_c \neg \mathit{Vehicle}\}$$
$$R^O = \{r_2 : \mathit{Vehicle}, \mathit{Park} \Rightarrow_O \neg \mathit{Enter},$$
$$r_{12} : \mathit{Vehicle}, \mathit{NarrowSpace} \Rightarrow_O \neg \mathit{Stop}\}$$
$$R^G = \{r_8 : \mathit{Bike}, \mathit{Park}, \mathit{Enter} \Rightarrow_G \textbf{fast\_circulation},$$
$$r_9 : \mathit{NarrowSpace}, \mathit{UnprotectedChildArea}, G\,\textbf{fast\_circulation} \Rightarrow_G \neg\textbf{pedestrian\_safety},$$
$$r_{13} : \mathit{NarrowSpace}, \mathit{Vehicle} \Rightarrow_G \textbf{fast\_circulation},$$
$$r_{10} : \mathit{Bike}, \mathit{Park}, \neg \mathit{Enter} \Rightarrow_G \textbf{pedestrian\_safety}\}$$
$$\succ = \{r_{11} \succ r_7\}$$
$$\mathscr{G} = \{\mathscr{G}(r_2) = \textbf{pedestrian\_safety}, \mathscr{G}(r_{12}) = \textbf{fast\_circulation}\}$$
$$> = \{\textbf{pedestrian\_safety} > \textbf{fast\_circulation}\}$$

Suppose *Enter* holds. This may correspond to a potential violation of r_2. This is not the case, because r_2 is not triggered and we do not derive *Vehicle*. However, we obtain the negation of the goal **pedestrian_safety** via r_9, i.e., the goal of r_2 is undermined. Condition 3 of the definition above is satisfied, since, via r_{10} the compliant situation ($\neg \mathit{Enter}$) promotes the goal of r_2.

Since we have r_7 we can construct a counts-as reasoning chain supporting *Vehicle*, and so the judge can expand the applicability conditions of r_2 by transforming it in the defeasible rule r_4:

$$r_4 : \mathit{Bike} \Rightarrow_c \mathit{Vehicle}$$

[8] Rules with free variables are interpreted as the set of all ground instances. Hence, we represent the facts and rules discussed in Section 3 as constituted by propositional literals.

By considering bicycles as vehicles we trigger r_{13} and promote **fast_circulation**, which is incompatible with **pedestrian_safety** (via r_9). However, **pedestrian_safety** is more important than **fast_circulation**.

The remaining case concerns contracting the applicability conditions of a norm in the cases illustrated in Table 1.

Definition 13 (Rule Contraction). *Let*

$$D = (F, G, R^c, R^O, R^G, \succ, \mathscr{G}, >)$$

be a normative theory, $r : b_1, \ldots, b_n \hookrightarrow_O l \in R^O$ *be a legal rule, and* $H = \{f_1, \ldots, f_m\}$ *be a context. If*

1. *D and $H \cup \sim l$* violate *the rule r, and*
2. *either*
   - *Case 8 of Table 1:*
     (a) *D and $H \cup \{\sim l\}$, and D and $H \cup \{l\}$, violate the goal g of r, and*
     (b) *there exists a $b_k \in A(r)$, such that b_k occurs in every goal reasoning chain $\mathscr{C}$ for $\neg g$ in at least one of the following normative theories*

$$(F \cup H \cup \{\sim l\}, G, R^c, R^O, R^G, \succ, \mathscr{G}, >) \qquad \text{if } \sim l \text{ is in } \mathscr{C}$$
$$(F \cup H \cup \{l\}, G, R^c, R^O, R^G, \succ, \mathscr{G}, >) \qquad \text{if } l \text{ is in } \mathscr{C}$$

   - *Case 10 of Table 1:*
     (a) *D and $H \cup \{\sim l\}$ promote the goal g of r, and D and $H \cup \{l\}$ violate the goal g of r,*
     (b) *there exists a $b_k \in A(r)$, such that b_k occurs in every goal reasoning chain $\mathscr{C}$ for $\mathbf{X}$ in at least one of the following normative theories*

$$(F \cup H \cup \{\sim l\}, G, R^c, R^O, R^G, \succ, \mathscr{G}, >) \qquad \text{if } \sim l \text{ is in } \mathscr{C},\ \mathbf{X} = g$$
$$(F \cup H \cup \{l\}, G, R^c, R^O, R^G, \succ, \mathscr{G}, >) \qquad \text{if } l \text{ is in } \mathscr{C},\ \mathbf{X} = \neg g$$

   - *Case 11 of Table 1:*
     (a) *D and $H \cup \{\sim l\}$, and D and $H \cup \{l\}$, promote the goal g of r,*
     (b) *there exists a $b_k \in A(r)$, such that b_k occurs in every goal reasoning chain $\mathscr{C}$ for g in at least one of the following normative theories*

$$(F \cup H \cup \{\sim l\}, G, R^c, R^O, R^G, \succ, \mathscr{G}, >) \qquad \text{if } \sim l \text{ is in } \mathscr{C}$$
$$(F \cup H \cup \{l\}, G, R^c, R^O, R^G, \succ, \mathscr{G}, >) \qquad \text{if } l \text{ is in } \mathscr{C}$$

then the contraction of the applicability conditions of a legal rule r with respect to the context H corresponds to the following operation $D^-_{b_k}$ over D:

$$D^-_{b_k} = (F, G, R'^c, R^O, R^G, \succ', \mathscr{G}, >)$$

where

$$R'^c = R^c \cup \{r : f_1, \ldots, f_m \rightsquigarrow \sim b_k\}$$
$$\succ' = \succ - \{s \succ r \mid r \in R'^c - R^c\}.$$

– *such that*

$$D' = (F \cup H, G, R'^c, R^O, R^G, \succ', \mathscr{G}, >) \vdash -\partial_G \neg g'$$

where g' *is the goal of any rule* $z \in R^O_{\mathrm{sd}}$ *applicable in* D' *such that* $g \not\succ g'$.

Remark 6. The applicability conditions of a legal rule should be contracted if the rule is applicable in the current context and it is violated by $\sim l$ according to Definition 9 (1) and one of the conditions 8, 10 and 11 of Table 1 is satisfied (2).

In Case 8 we have that the goal is violated not only by $\sim l$ but also by complying with the norm (a); moreover, there exists at least one of the antecedents of the legal rule which is used in all goal reasoning chains, in which either l or $\sim l$ occur, to prove $\neg g$ (the goal violation of the legal rule) (b). Thus, we have reasons to block the counts-as derivation of such an antecedent.

In Case 10, we have that the violation of the norm ($\sim l$) unexpectedly promotes the goal g of the norm, while compliance (l) doesn't (a); moreover, there exists at least one of the antecedents of the legal rule which is used in all goal reasoning chains, in which l occurs, to prove g (the goal promotion of the legal rule), while an antecedent is used in all goal reasoning chains, in which $\sim l$ occurs, to prove $\neg g$ (the goal violation of the legal rule) (b). Thus, we have reasons to block the counts-as derivation of such antecedents.

Finally in Case 11, we have that the goal of the norm is promoted independently from the fulfilment or violation of the norm (a); moreover the same as part (b) of Case 8 must hold.

If (1) and (2) hold, the normative theory can be contracted by adding defeaters to the rules which can make true the condition b_k identified in the (b) steps of the cases above.

As for the case of expansion, the resulting normative theory is however subject to the final constraint that no other more important goal g' of any norm would be violated ($-\partial_G \neg g'$).

Example 4. Let us simplify the normative theory of the Example 3 to illustrate Case 10 of Table 1. (The other cases of contraction in the table have a similar treatment.)

$$\begin{aligned}
F &= \{Enter\} \\
H &= \{Bike, Park\} \\
G &= \{\textbf{fast_circulation}\} \\
R^c &= \{r'_7 : Bike \Rightarrow_c Vehicle\} \\
R^O &= \{r_2 : Vehicle, Park \Rightarrow_O \neg Enter\} \\
R^G &= \{r_8 : Bike, Park, Enter \Rightarrow_G \textbf{fast_circulation}, \\
&\qquad r_{14} : Park, Vehicle, \neg Enter \Rightarrow_G \neg\textbf{fast_circulation}\} \\
\succ &= \{r_{14} \succ r_8\} \\
\mathscr{G} &= \{\mathscr{G}(r_2) = \textbf{fast_circulation}\} \\
> &= \emptyset
\end{aligned}$$

The facts in F and H make all rules applicable, with the exception of r_{14}. The violation of r_2, which allows for deriving $+\partial^O \neg Enter$, leads to promote the goal of this rule, while the compliance implies the violation of the goal. Hence, conditions 1 and (a) of Case 10 in Table 1 are satisfied. The only literal we can contract is *Vehicle*, which occurs in the goal reasoning chain supporting ¬**fast_circulation** (*Bike* and *Park* are facts and cannot be removed). Thus, we can contract the applicability conditions of r_2 by adding a defeater $r_{15} : Bike, Park \leadsto_c \neg Vehicle$ and by stating that r_{15} is stronger than r'_7.

Note that the operations $D^*_{b_k,\ldots,b_{k+j}}$ and $D^-_{b_k}$ introduced in Definitions 12 and 13 correspond to special cases of AGM revision and contraction of conclusions in DL [3]. Indeed, under some preconditions, expanding the applicability conditions of a norm amounts to modifying the rules and the superiority relation even if the negation of one or more elements in $b_k,\ldots,b_{k+j}$ are derivable in D. However, due to the sceptical nature of DL, we still do not get a contradiction. On the other hand, under suitable preconditions, contracting the applicability conditions of a norm corresponds to preventing the proof of b_k. R'^c ensures that if b_k has been proven, a defeater with head $\neg b_k$ will fire. [3] provided a reformulation within DL of AGM postulates for revision and contraction: the results provided there can be trivially extended to our framework

Theorem 2. *If preconditions 1, 2, 3, and 4 of Definition 12 hold, $D^*_{b_k,\ldots,b_{k+j}}$ satisfies the reformulation of AGM postulates for revision given in [3]. If preconditions 1 and 2 of Definition 13 hold, $D^-_{b_k}$ satisfies the reformulation of AGM postulates for contraction given in [3].*

6 Related Work and Conclusions

In this paper we proposed a formal method for modelling extensive and restrictive interpretations in statutory law. The contribution is based on the idea that the interpretation of legal concepts may require to change the counts-as rules defining them. Indeed, if our ontology does not classify a bike as a vehicle, but we have reasons that this is the case, then this implicitly leads to conclude that the ontology must be revised and that a bike, at least in the contexts under consideration, is a vehicle. In this paper we assume that a reason is a chain of rules (or better a tree) a set of premises (corresponding to the context) to the conclusion we want to support, and that we can revise it if there are defeaters in the chain that prevent a positive derivation of the conclusion. The operation we perform for expansion is to strengthen such defeaters to defeasible rules, while for contractions we simply introduce new defeaters. In the expansion case, the transformation of defeaters in defeasible rules allows us to derive the desired conclusion; in the case of contraction the new defeaters prevent its derivation, but at the same time they do not allow for the derivation of the opposite. The technique presented here is not the only possible; another option, not considered in this paper is to look again at chain of rules, and instead of changing the strength of the rules we can change the superiority relation (i.e., the relative strength of rules), as proposed in [9].

This revision operation presented in this paper are driven and constrained by considering the goal of the legal rules in which these concepts occur. To the best of our knowledge, there is no work so far devoted to the dynamics and revision of constitutive rules, and no proposal regarding how to model the interpretation of legal rules in these terms. In our perspective, it is possible to identify some interesting logical links between the process of legal interpretation and AGM operations of contraction and revision in rule-based systems such as DL (Theorem 2). In addition, our work has the advantage of using a logical framework which is computationally feasible (see Theorem 1).

An extensive literature is devoted to legal ontologies (see, e.g., the survey in [6]), but it is oriented to develop applications in the field of semantic web and rule interchange languages for the legal domain, applications which are not our primary concern. Also, these approaches usually fail to deal with the defeasibility and dynamics of legal concepts.

The possibility to model legal and normative ontologies via constitutive rules has a solid philosophical backgrounds (see [20,19]). In the field of normative multi-agent systems, the only work which addressed the problem of the penumbra of legal concepts within a complete theory of counts-as rules is [13]. [13] provides very complex modal account of counts-as rules in which the problem of the penumbra is analysed here in terms of the notion of context. A 'penumbral meaning' is then nothing else but the set of individuals on which the contextual interpretation of the concept varies. However, [13] does not explain how the different extensions of a concept are related to the contexts depending on the regulative norm whose violation is discussed. In addition, what is lacking in that work is that it does not address the problem of the dynamics of constitutive rules and does not consider the role of normative goals in determining the applicability conditions of legal rules.

Several works in the literature of AI & Law have considered the role of teleological reasoning in the legal interpretation. Indeed, this idea is standard in legal theory and the goals of legal rules are recognised by jurists as decisive in clarifying the scope of the legal concepts that qualify the applicability conditions for those rules [2,17,21,14]. [2,17] use goals and values in frameworks of case based reasoning for modelling precedents mainly in a common law context. [21] analyses a number of legal arguments even in statutory law, which include cases close to the ones discussed here. The proposal which is closer to our contribution is [14]. In [14] Jaap Hage addresses, among others, the problem of reconstructing extensive and restrictive interpretation. This is done in Reason-Based Logic, a logical formalism that can deal with rules and reasons: the idea is that the satisfaction of rules' applicability conditions is usually a reason for application of these rules, but there can also be other (and possibly competing) reasons, among which we have the goals that led the legislator to make the rules.

All these approaches in AI & Law highlight the importance of rule goals, and [14], in particular, follows this idea to formalise extensive and restrictive interpretation. However, it seems that no work so far has attempted to couple this view with a framework for reasoning with counts-as rules and their dynamics. In this perspective, we believe that this paper may contribute to fill a gap in the literature.

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